Intellectual Property Rights Protection and Endogenous Economic Growth

by

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Abstract

The main purpose of the paper is to examine the impact of intellectual property rights (hereinafter IPR) protection on economic growth and welfare. To achieve this aim, we make use of an expanding-variety type R&D-based endogenous growth model. We work out the transitional dynamics of a shock in IPR protection and account fully for the static losses and dynamic gains of tightening IPR protection. We find that there does exist such an optimal degree of IPR protection in our model. We then calibrate our model by US data, and found that under-protection of IPR is much more likely than over-protection. Moreover, in the case of over-protection, the welfare losses are trivial; whereas in the case of under-protection, the welfare losses can be substantial.

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1 Introduction

The main purpose of the paper is to examine the impact of intellectual property rights (hereinafter IPR) protection on economic growth and welfare. To achieve this aim, we make use of an expanding-variety type R&D-based endogenous growth model a la Romer (1990) and Rivera-Batiz and Romer (1991). The simplest way to model IPR protection is to assume that imitation is costless, and that stronger IPR protection lowers the rate of imitation. This is the approach we adopt here.[1]

The conventional wisdom in the literature about strengthening IPR protection is that there is a tradeo® between static losses and dynamic gains, so that there is a possibility of the existence of an optimal degree of IPR protection. The existence and properties of such an optimum, however, have not been con¯rmed before in the endogenous growth literature, partly because it involves a full characterization of the transitional dynamics of the rate of innovation and fraction of goods imitated in the economy. If transitional dynamics are not considered, and one focuses only on the steady state, then the welfare analysis is misleading. In fact, steady state welfare is maximized when growth rate of consumption is maximized. This will be achieved by protecting IPR fully and forever. Obviously, such a corner solution is intuitively unappealing and also counter-factual, since the transitional welfare gains and losses are not taken into account. This paper shows that once transitional dynamics are taken into account, there exists a ¯nite optimal degree of IPR protection.

One contribution of our paper is that we work out the transitional dynamics of a shock in IPR protection and account fully for the static losses and dynamic gains of a tightening of IPR protection. Speci¯cally, we ¯nd that when the government announces an immediate increase in IPR protection, there is an immediate drop in the entire consumption path and an immediate increase in the rate of growth of consumption, as well as overshooting of the rate of innovation. The static losses therefore come from the immediate shift in consumption level, while the dynamic gains come from the immediate shift in the rate of growth of consumption. Stronger IPR protection reduces the rate of imitation and increases the average duration of monopoly power of each innovator. The drop in consumption is due to the fact that the increase in average duration of monopoly power of each good leads to an increase in the average price of goods in the economy, leading to lower demand for goods in the aggregate. The acceleration in consumption growth comes from the fact that longer average duration of

[1] If we assumed that imitation is costly, then tightening IPR protection amounts to increasing the cost of imitation. In that case, we believe similar results would obtain.

monopoly power leads to higher expected pro¯ts for all innovators, which encourages more entry into innovation, and hence higher rate of innovation and higher rate of growth. Our model draws from the `laboratory equipment' version of Romer (1990), while the dynamic analysis is similar to that of Helpman (1993).

To assess quantitatively the welfare signi¯cance of optimal IPR protection, we calibrate our model by US data about long-term growth rate, mark-up factor in manufacturing industries, time rate of preference and intertemporal elasticity of substitution. The calibration results indicate that there is under-protection of IPR (relative to the optimal level) within plausible range of parameter values, and that under-protection of IPR is much more likely than over-protection. More complete computation indicates that in the case of over-protection, the welfare losses are trivial; whereas in the case of under-protection, the welfare losses can be substantial. One interpretation of this result is that the US should protect IPR much more than it currently does.

There are by and large two types of R&D-based endogenous growth models: expanding-variety type and quality-ladder type. O'Donoghue and Zweimuller (1998) construct a quality-ladder type R&D-based endogenous growth model in the tradition of Grossman and Helpman (1991) and Aghion and Howitt (1992). They merge the patent-design literature and endogenous-growth literature incorporating both length and breadth of patent in the quality ladder. They point out the short-comings of the partial equilibrium patent-design analyses, which omit the general equilibrium e®ects. One of these e®ects is that when multiple industries use patent protection, the monopoly distortion e®ect can be greatly diminished. In our model, however, such monopoly e®ect is central to the static-dynamic tradeo® when IPR protection is strengthened across industries. Futagami, Mino and Ohkusa (1996) study optimal patent length in a Grossman-Helpman type quality-ladder model. Although they identify an optimal patent length under certain conditions, there is no transitional dynamics as in our model. Nonetheless, their work is an interesting complement to our paper.

Section 2 lays out the model, and derive the dynamics when there is an immediate increase of IPR protection. The optimal degree of IPR protection is derived. In section 3, we calibrate the model to the US economy. Since closed form solution is not possible, we solve the dynamic general equilibrium numerically, and compute the optimal degrees of IPR protection that correspond to di®erent assumed actual monopoly durations of the innovators. Section 4 concludes with some discussion on future extensions.

2 The Model

The model is a dynamic general equilibrium one, with expanding-variety type R&D as the engine of growth. There is only one final good, which can be used for consumption, for production of intermediate goods, and for R&D, which is needed to invent new varieties of intermediate goods. The production function for the final good is characterized by an expanding variety of producer intermediates of the Dixit-Stiglitz (1977) form:

$$Y = L^{1-\alpha} \int_0^A x(i)^{\alpha} di; \qquad 0 < \alpha < 1 \tag{1}$$

where Y is the quantity of final good; L is labor input; $x(i)$ is the variety of producer intermediates with index i; and A; the number of varieties, increases over time as a result of innovations. The final good market is perfectly competitive.

The intermediate good market is monopolistically competitive a la Dixit-Stiglitz (1977), Ethier (1982) and Romer (1990). Sellers are innovators of intermediate goods and buyers are final good producers. There is no uncertainty in innovation. Motivated by the prospect of monopoly profit, an innovator invests in β units of final good and obtains a blueprint of a new variety. It then earns the opportunity to produce the new intermediate good at unit marginal cost (i.e., the cost of one unit of final good) and sell the differentiated intermediate good at a profit-maximizing markup of $1/\alpha$.

To allow a role for IPR protection, following Helpman (1993), we assume an imitation process of the form

$$\dot{A}_c = \mu(A - A_c); \qquad \mu > 0 \tag{2}$$

The variable A_c is the number of goods that have been imitated; whereas $A - A_c$ is the number of goods that have not been imitated and thus available for imitation. The parameter μ captures the strength of IPR protection, with higher value meaning weaker protection. It is the hazard rate at which the market power of an intermediate good producer disappears at the next date, given that its market power has not been eroded so far. This rate is defined as the rate of imitation. The rate of imitation is dependent on many factors, e.g., natural rate of imitation (the rate of imitation when there is no IPR protection at all), IPR protection (patent length and breadth, trademark, copyrights, enforcement of IPR, etc), use of trade secrets, use of masquerading to prevent imitation, etc. One way to capture explicitly these effects is to decompose μ into two terms: $\mu \equiv \iota\delta$, where ι is the natural rate of imitation for some given degree of usage of trade of secrets, masquerading, etc., and $0 < \delta < 1$ is an index of the strength of IPR protection provided by the government, with higher δ representing weaker protection. Obviously, full IPR protection implies that $\delta = 0$, and no IPR protection implies that $\delta = 1$. Hereinafter, we shall refer to a tightening of IPR protection as a decrease in μ (caused by a decrease in δ).

Although μ is influenced by an array of factors, in the rest of this paper, we shall treat other factors as constant and regard μ as a parameter that can be controlled by the government through its IPR policy.

Once a product is imitated, we assume that competition will drive the price down to marginal cost. Thus, we can classify the intermediate goods into two groups: goods with index $i \in (0; A_c)$ are the imitated ones that are competitively priced, and the rest, with index $i \in (A_c; A)$, that are still under monopoly. The demand functions for the two groups are

$$x(i) = \begin{cases} L\alpha^{1/(1-\alpha)} \equiv x_c; & i \in (0; A_c) \\ L\alpha^{2/(1-\alpha)} \equiv x_m; & i \in (A_c; A) \end{cases} \tag{3}$$

Clearly, $x_m < x_c$, which reflects the usual monopoly distortion in resource allocation. It follows that the resource constraint for the economy can be written as

$$Y = C + \beta A + A_c x_c + (A - A_c) x_m \tag{4}$$

where C is aggregate consumption.

Taking into account μ and the instantaneous profit at each future date, a potential innovator decides whether or not to enter into the innovation business. Under the assumption of free entry into the innovation business, the present discounted value (PDV) of net profits of an innovator is equal to zero in equilibrium. That is, the rate of return to innovation, r_m, must be equal to the real interest rate adjusted for imitation risk:

$$r_m = r + \mu \tag{5}$$

From (3) it follows that the rate of return $r_m = (L/\beta)\alpha^{2/(1-\alpha)}(1/\alpha - 1)$. The value of a firm equals to the cost of innovation if there are no barriers to entry in the innovation business. Therefore, the PDV of the net profits of a firm is zero. If there are entry barriers in the innovation business, the PDV of net profits of an innovator is positive. The higher the barriers, the larger the PDV of net profits.

The representative consumer, who also owns the firms, is assumed to choose a consumption path $c(t)$ to maximize the utility function

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} dt; \quad \theta > 0 \tag{6}$$

subject to the usual life-cycle budget constraint with asset value equal to the value of the firms. Applying standard optimal control arguments and making use of (5), we can write the consumer optimality condition as

$$\gamma_c \equiv \frac{\dot{C}}{C} = \frac{1}{\theta}(r_m - \mu - \rho) \tag{7}$$

2.1 Transitional Dynamics

Define $g \equiv A_c/A$ and $h \equiv C/(\beta A)$. The variable g is the fraction of goods that have been imitated and thus $g \in [0; 1]$. The variable h is a scaled and normalized version of consumption C. Using (1), (2), (3), (4) and (7), it can be shown that the dynamics of the market equilibrium can be summarized by two differential equations:

$$\begin{cases} \dot{g} = \mu(g^{-1} - 1) + \gamma_1 g + \gamma_2 h + \gamma_3 \\ \dot{h} = \gamma_c + \gamma_1 g + \gamma_2 h + \gamma_3 \end{cases} \tag{8}$$

where $\gamma_1 \equiv -r_m \left[\alpha^{-1/(1-\alpha)} - \alpha^{-1} - 1\right] < 0$; $\gamma_2 \equiv 1$; $\gamma_3 \equiv -r_m(1+\alpha)/\alpha < 0$.[2] Equations (8) is an ordinary differential equation system of which a stable solution is determined by an initial condition at $t = 0$ and a boundary condition at $t = \infty$: The boundary condition is given by the steady state

$$g^* = \frac{\mu}{\gamma_c + \mu}; \quad h^* = -\frac{\gamma_1}{\gamma_2} g^* - \frac{\gamma_c + \gamma_3}{\gamma_2} \tag{9}$$

Notice that if $\mu = 0$, (8) is linear and admits a closed form solution. Figure 1 depicts the system's phase diagram which summarizes the transitional dynamics. The two curves corresponding to $\dot{g} = 0$ and $\dot{h} = 0$ always intersect, though not necessarily at positive h. A sufficient condition for the existence of a positive steady state is the $\dot{h} = 0$ curve having a positive intercept, i.e. $\gamma_c + \gamma_3 < 0$. The phase diagram reveals that the dynamic system is saddle-path stable. Along the stable arm, if the economy starts from point X at which g

[2] Although γ_1, γ_2 and γ_3 are independent of μ, changes in μ have both a static effect (level effect) and a dynamic effect (growth effect). See Section 2.2 for more detail.

and h are below the steady state, both g and h will rise monotonically along the transitional path. Similarly, starting from point Y at which g and h are above the steady state, both g and h decline monotonically during the transition.

[Insert Figure 1 here]

2.2 Comparative dynamics around the steady state

We can learn more about the transitional dynamics by linearizing the differential equation system around the steady state. The linearized version can be written as

$$\begin{bmatrix} \dot{g} \\ \dot{h} \end{bmatrix} = M \cdot \begin{bmatrix} g - g^* \\ h - h^* \end{bmatrix} \tag{10}$$

where M is a 2 x 2 matrix with the (i,j) element a_{ij} being

$$a_{11} = -\mu \cdot (g^*)^{-2} + \gamma_1 < 0; \quad a_{12} = a_{22} = \gamma_2 > 0; \quad a_{21} = \gamma_1 < 0. \tag{11}$$

Let λ_1 and λ_2 be the two eigenvalues of M. Since $\lambda_1 \lambda_2 = |M| = -\mu\gamma_2(g^*)^{-2} < 0$, λ_1 and λ_2 must be real and opposite in sign. This means that the dynamic system is saddle-path stable, confirming the qualitative conclusion of the phase diagram. Solving the characteristic equation $|M - Ix| = 0$, the two eigenvalues are

$$\lambda_1 = \frac{1}{2}[(a_{11} + a_{22}) + B^{1/2}]; \quad \lambda_2 = \frac{1}{2}[(a_{11} + a_{22}) - B^{1/2}] \tag{12}$$

where $B = (a_{11} + a_{22})^2 - 4|M| > 0$. Since the two eigenvalues are of opposite sign, it follows that $\lambda_1 > 0$ and $\lambda_2 < 0$. The general solution of the linearized system is

$$\begin{cases} g(t) - g^* = b_1\nu_{11}e^{\lambda_1 t} + b_2\nu_{12}e^{\lambda_2 t} \\ h(t) - h^* = b_1\nu_{21}e^{\lambda_1 t} + b_2\nu_{22}e^{\lambda_2 t} \end{cases} \tag{13}$$

where $[\nu_{1i} \quad \nu_{2i}]'$ is the eigenvector corresponding to λ_i, $i = 1, 2$, and b_1 and b_2 are constants to be determined by boundary conditions. Using the initial condition $g(0)$ and the asymptotic boundary condition $g(\infty) = g^*$, which characterizes the stable saddle-path, it follows that $b_1 = 0$ and $b_2 = g(0) - g^*$. Normalize $\nu_{12} = 1$ and write $\nu_{22} \equiv \nu$ and $\lambda_2 \equiv \lambda < 0$; we have

$$\begin{cases} g(t) = g^* + [g(0) - g^*]e^{\lambda t} \\ h(t) = h^* + [g(0) - g^*]\nu e^{\lambda t} \end{cases} \tag{14}$$

Let us determine the sign of ν. By definition,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\ \nu \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ \nu \end{bmatrix} \tag{15}$$

Solving for ν, and using (11) and (12), it follows that

$$\nu = (\lambda - a_{11})/a_{12} = -a_{21}/(a_{22} - \lambda) > 0 \tag{16}$$

By combining the two equations in (14), we obtain the "policy function" $h(g)$ (as in dynamic programming) which is nothing but the equation for the stable saddle-path on the phase diagram:

$$h(g) = (h^* - \nu g^*) + \nu g \tag{17}$$

Thus, around the steady state, $h'(g) = \nu > 0$ so that the stable saddle-path is upward sloping, confirming what we have found from the phase diagram. Now, let us evaluate the impact of a change in μ on the paths of g and h. Using (7) and (9), it is straightforward to check that both g^* and h^* increase with μ:

$$\frac{\partial g^*}{\partial \mu} = \frac{\theta\gamma_c + \mu}{(\gamma_c + \mu)^2\theta} > 0; \quad \frac{\partial h^*}{\partial \mu} = -\frac{\gamma_1}{\gamma_2}\frac{\partial g^*}{\partial \mu} + \frac{1}{\theta\gamma_2} > 0 \tag{18}$$

For analytical tractability, following Helpman (1993), we consider the first order response of $(g; h)$ to changes in μ by differentiating (14) with respect to μ, while ignoring the impact of μ on λ and ν:

$$\frac{\partial g(t)}{\partial \mu} = (1 - e^{\lambda t})\frac{\partial g^*}{\partial \mu} \geq 0 \tag{19}$$

$$\frac{\partial h(t)}{\partial \mu} = \frac{\partial h^*}{\partial \mu} - \nu e^{\lambda t}\frac{\partial g^*}{\partial \mu} = F(t) \cdot \frac{\partial g^*}{\partial \mu} + \frac{1}{\theta\gamma_2} > 0; \tag{20}$$

where $F(t) \equiv \{-\gamma_1\gamma_2(1 - e^{\lambda t}) + \gamma_1\lambda\}/\{\gamma_2(\gamma_2 - \lambda)\} > 0$. In particular, at $t = 0$, $\partial g(0)/\partial\mu = 0$, implying that there is no jump in g as μ decreases (IPR protection tightens). However, $\partial h(0)/\partial\mu > 0$, which means that there is a downward jump in h as μ decreases. On the phase diagram, such a downward jump of the initial h shows up as a downward shift of the entire stable saddle-path as illustrated in Figure 2. Suppose we start from X on the saddle-path $h(g; \mu)$ corresponding to a certain value of μ. Now suppose IPR protection is tightened so that $\mu \downarrow \mu'$. The value $h(0) \equiv h(g(0); \mu)$ is no longer on the equilibrium path; rather, the equilibrium initial h should take a discrete downward jump from X to Y, with the size of the jump given by $\partial h(0)/\partial \mu$. Since X is arbitrary, this implies that the entire saddle-path must shift downward as shown. Alternatively, the downward shift of the saddle-path can be discerned by differentiating the policy function (17) with respect to μ. Note that the entire saddle-path is changed as μ changes, as shown in Figure 2. The downward jump of C at $t = 0$ cannot be accounted for without solving for the entire new equilibrium saddle-path.

[Insert Figure 2 here]

For $t > 0$, we see that $\partial g(t)/\partial \mu > 0$ and $\partial h(t)/\partial \mu > 0$. Thus, following a fall in μ (tightening IPR protection), both g and h fall at each point in time and converge to the new steady state. Figure 3 illustrates the comparative dynamics on the phase diagram, whereas Figures 4 and 5 show the time paths of g and h as IPR is tightened. In this model, the transitional dynamics of $(g; h)$ is monotone, unlike Helpman (1993) in which the transitional adjustment may be non-monotone.

[Insert Figures 3 - 5 here]

We can say more about the impact of changes in μ on the innovation rate $\dot{A}/A$. Starting from (4) and making use of (1) and (3), we can write

$$\frac{\dot{A}}{A} = -\gamma_1 g - \gamma_2 h + \beta_3 \tag{21}$$

where $\beta_3 \equiv (L/\beta)\alpha^{2\alpha/(1-\alpha)}(1-\alpha^2) > 0$. Differentiating (21) with respect to μ and making use of (18), (19), (20), and (16), it can be shown that

$$\frac{\partial}{\partial \mu}\left(\frac{\dot{A}}{A}\right) = H(t) - \frac{1}{\theta} < 0; \quad \text{where} \tag{22}$$

$$H(t) \equiv -\left[\frac{\partial g^*}{\partial \mu}\,\frac{\gamma_1 \lambda}{\gamma_2 - \lambda}\right] e^{\lambda t} < 0$$

Clearly, $H(t) \uparrow 0$ as $t \to +\infty$. This implies that there will be an initial over-shooting of the innovation rate $\dot{A}/A$ as μ is lowered (IPR protection is tightened). As t increases, $\partial(\dot{A}/A)/\partial \mu$ gradually approaches the long run value $-1/\theta = \partial \gamma_c/\partial \mu$. Figure 6 depicts the dynamic adjustment path of the innovation rate in response to tightened IPR protection. The economy starts out at a steady state at which the innovation rate is equal to $\gamma_c(\mu)$, given a certain level of IPR protection corresponding to μ. The new, tightened IPR protection level (with the corresponding $\mu' < \mu$) implies a higher steady state growth rate $\gamma_c(\mu') = \gamma_c(\mu)+1/\theta$. The innovation rate initially overshoots by the amount $|H(0)|$ and then gradually converges to the new steady state growth rate.

[Insert Figure 6 here]

2.3 Tightening IPR Protection — Static Loss vs. Dynamic Gain

Tightening IPR protection will induce an immediate loss of current consumption arising from more monopoly distortion, but a gain in future consumption as a result of faster innovation and faster growth. More precisely, there will be a downward level shift of the entire consumption path, but the path will become steeper as a result of faster growth. We have already seen the consumption level shift in Figure 2, because $h \equiv C/(\beta A)$ is nothing but a scaled and normalized version of consumption C. To see more clearly the tradeoff between current and future consumption, let us find out the equilibrium consumption path. The consumer optimality condition (7) requires that equilibrium consumption grows at the rate $\gamma_c[\mu] = (r_m - \mu - \rho)/\theta$ which depends negatively on μ: $C(t) = C(0)\exp(\gamma_c[\mu]t)$. Rewrite $h(t)$ as $h[t;\mu]$ to emphasize its dependence on μ, by definition of h, we can write the equilibrium consumption path as $C(t) = \beta A(0)h[0;\mu]\exp(\gamma_c[\mu]t)$. Taking logarithm and differentiating with respect to μ, we have

$$\frac{\partial \ln C(t)}{\partial \mu} = \frac{1}{h(0)}\frac{\partial h[0;\mu]}{\partial \mu} + \frac{\partial \gamma_c[\mu]}{\partial \mu}\cdot t \qquad (23)$$

The first term on the right hand side of (23) measures the extent of the consumption level shift at $t = 0$, where $\partial h[0;\mu]/\partial \mu > 0$ is given by (20) above. Such a level shift in the consumption path is the static loss from tightening IPR protection. Since $\partial \gamma_c[\mu]/\partial \mu = -1/\theta$, the second term on the right hand side of (23) measures the steepening of the consumption path, which represents the dynamic gain from tightening IPR protection. Figure 7 displays the time path of $C(t)$ before and after a tightening of IPR protection is announced.

[Insert Figure 7 here]

Transitional dynamics in the growth rate of C exist if we used a more sophisticated imitation function than (2). The results with such a function are reported by the authors elsewhere. Introduction of transitional dynamics in the growth rate of C does not change the results in this paper in any major way. In may appear that, since the new steady state growth rate is attained immediately, it is straightforward to calculate the welfare and therefore the optimal IPR analytically. However, the static loss, namely the one-shot fall in $C(0)$, cannot be obtained unless the entire new saddle-path is calculated, which can be done exactly only by numerical method.

2.4 Optimal IPR protection

The tradeoff between static loss and dynamic gain in consumption naturally leads to the notion of optimal IPR protection — optimal in the sense of maximizing the representative agent's utility. In our model this amounts to choosing $\mu = \mu^*$ such that utility (6) is maximized, subject to the law of motion of equilibrium consumption. As shown above, the equilibrium consumption path is $C(t) = \beta A(0)h[0;\mu]\exp(\gamma_c[\mu]t)$. Substituting $c(t) \equiv C(t)/L$ into the utility function (6), evaluating the relevant integrals and taking logarithm, the optimal μ can be characterized by

$$\mu^* = \arg\max_{\mu} \begin{cases} s(1-\theta)\ln h[0;\mu] - s\ln(\rho - \gamma_c[\mu](1-\theta)); & \theta \neq 1 \\ \ln h[0;\mu] + \gamma_c[\mu]/\rho & \theta = 1 \end{cases} \tag{24}$$

where

$$s = \begin{cases} 1 & \text{if } 1-\theta > 0 \\ -1 & \text{if } 1-\theta < 0. \end{cases} \tag{25}$$

Notice that $\rho - \gamma_c(1-\theta) > 0$ as dictated by the transversality condition from the consumer optimal control problem. Referring to Figure 3, different μ corresponds to different saddle-path with the corresponding $h(g_0;\mu) = h[0;\mu]$, as indicated by points X, Y, and Z. The trailing terms in (24), $-s\ln(\rho - \gamma_c[\mu](1-\theta)$ and $\gamma_c[\mu]/\rho$, reflect the utility of the consumption path from a starting point (say Z) to the corresponding steady state (say Z'). For an interior optimum, μ^* satisfies

$$\frac{\partial \ln h[0;\mu^*]}{\partial \mu} = -\left(\frac{1}{\rho - \gamma_c[\mu^*](1-\theta)}\right)\frac{\partial \gamma_c[\mu^*]}{\partial \mu} \qquad \text{for all } \theta > 0 \tag{26}$$

We have shown that the left hand side is greater than zero. Also, $\partial\gamma_c[\mu^*]/\partial\mu = -1/\theta < 0$, so the right hand side is greater than zero too. In fact, we can call the left hand side the marginal costs and the right hand side the marginal benefits of tightening IPR protection. The marginal costs come from a decrease in current consumption, and the marginal benefits come from an increase in consumption growth, which results in higher utility in the future.

3 Model calibration

In this section we calibrate the model to the US economy to get some idea about its practical relevance. In particular, we are interested in solving for μ^*, which parameterizes the optimal IPR protection level, and quantifying its welfare implications. As can be seen from (24), the objective function depends on $h[0;\mu]$ which is nothing but the policy function (i.e. the stable saddle-path) evaluated at $g(0)$. Rather than relying on the approximate solution in (17), we solve for the policy function numerically from the differential equation system (8) in our calibration exercise. Readers not interested in the numerical computation may skip the following section without loss of continuity.

3.1 Numerical solution

By taking the ratio of the two differential equations in (8), we obtain a differential equation in $(g;h)$ space which characterizes the two saddle-paths (one stable, one unstable) of the system:

$$h'(g) \equiv \frac{dh}{dg} = \frac{\dot{h}}{\dot{g}} = \frac{\gamma_1 g + \gamma_2 h + \gamma_c + \mu + \gamma_3}{\mu g - \mu + \gamma_1 g + \gamma_2 h + \gamma_3} \equiv \frac{H(g)}{G(g)} \tag{27}$$

with initial value $h(g^*) = h^*$. The last identity gives the definitions of $H(g)$ and $G(g)$. The two saddle-paths are distinguished by their slopes at g^*, but (27) alone is not sufficient to pin down the stable path because $h'(g^*) = H(g^*)/G(g^*) = 0/0$. To calculate the slope of the stable path around the steady state, we use the L'Hopital's rule to evaluate $h'(g^*) = H'(g^*)/G'(g^*)$, and then write it in terms of the coefficients a_{ij} of the linearized system (10) as:

$$h'(g^*) = \frac{a_{21} + a_{22}h'(g^*)}{a_{11} + a_{12}h'(g^*)} \tag{28}$$

The two roots of the quadratic equation (28) are

$$h'(g^*) = \frac{-(a_{11} - a_{22}) \pm \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2a_{12}} = \frac{\lambda_i - a_{11}}{a_{12}} > 0; \quad i = 1;2 \tag{29}$$

where the second equality can be verified by substituting (12) for λ_1 and λ_2, the two eigenvalues of the linearized system. From the phase diagram we know that the stable path is flatter than the unstable path, so that the required slope must be the smaller root. Since $\lambda_1 > 0$ and $\lambda_2 < 0$, it follows that the slope of the stable path is $(\lambda_2 - a_{11})/a_{12}$ which is simply ν, the second component of the normalized eigenvector corresponding to the negative eigenvalue of the linearized system (see (16)).

Standard numerical algorithms are available for solving (27) which is an initial value problem in ordinary differential equation (e.g. Press et al. (1992, Chapter 16)). We use numerical routines in module nag_ivp_ode_rk in NAG Fortran 90 library (release 3, 1998) which implements a variable-step Runge-Kutta method. In searching over the optimal μ, each evaluation of the objective function in (24) with respect to μ requires the differential equation (27) be solved once to yield $h[0;\mu]$. That is, the differential equation solver has to be embedded within the optimization routine. We perform the computation on a Pentium PC with programs written in Gauss and Fortran 90. The computer programs are available upon request for replication.

3.2 Calibration result

We calibrate the model with US data. To tie down the two preference parameters, θ (intertemporal substitution) and ρ (discount factor), we make use of the consumer optimality condition (7) with the steady state growth rate, γ_c, and the real interest rate, r, equated to the observed values in the post-war era. Following King et al (1988) we set $\gamma_c = 0.016$, the common trend annual growth rate of output, consumption and investment, and $r = 0.065$, the average real return to equity. Since both ρ and θ are positive, it follows that $0 < \theta < r/\gamma_c \approx 4$. θ is usually assumed to exceed one in the literature. We will consider $\theta \in [1;4]$ as in Stokey (1995). For each θ in the interval, the consumer optimality condition then implies a unique ρ.

We need the initial state $g(0)$ to compute $h[0;\mu] = h(g(0);\mu)$ in (24). Assuming the US economy is currently at a steady state, corresponding to a particular value μ_0 (to be determined later), we set $g(0) = g^* \equiv \mu_0/(\gamma_c + \mu_0)$ using (9). The imitation rate μ_0 is calibrated to reflect the IPR protection level currently in place. We relate μ_0 to the duration T of a finite-life monopoly:

$$\frac{1}{\mu_0 + r} = \int_0^T e^{-rt}dt \qquad (30)$$

The left-hand-side is the expected present value of a profit stream of \$1 for a perpetual monopoly that faces a hazard rate of imitation μ_0. The right hand side is the certainty equivalence (CE) of the same profit stream for a finite-life monopoly that lasts only up to T. In other words, T is the certainty-equivalent duration of a monopoly position, given the risk of being imitated as indicated by μ_0. Given $r = 0.065$, we can assign a value for T (in years) and then back out the corresponding μ_0. The advantage of this approach is that we have better idea about the plausible range of T than μ_0. For example, $T = 17$ years (the current patent length in US) corresponds to $\mu_0 = 0.032$.

The technology parameter α has two interpretations: labor share $1 - \alpha$ and markup ratio $1/\alpha$. For the purpose of this paper, the markup interpretation seems to be more appropriate. The labor share interpretation, however, has the advantage of allowing us to fix α almost unambiguously by using the conventional labor share value of 0.6, which implies a markup ratio of 2.5. Hall (1986) estimates the markup ratios of some fifty industries at the two-digit SIC code level, covering all sectors of the US economy, and concludes that in most industries the markup ratio is above 1.5 and in a few it exceeds 3. We will consider markup ratio $1/\alpha \in [1.25, 2.5]$ and use Hall's estimate of 1.6 for the whole manufacturing industry as the benchmark value. The remaining unknown parameter is the ratio L/β. Given r and μ_0, the zero profit (no arbitrage) condition determines the rate of return $r_m = r + \mu_0$. The ratio L/β can then be solved from the expression $r_m = (L/\beta)\alpha^{2/(1-\alpha)}(1/\alpha - 1)$.

Table 1 reports the optimal IPR protection levels and their welfare implications for three assumed levels of IPR protection currently in place (i.e., three values of μ_0 that correspond to 5, 10 and 17 years of CE monopoly duration, respectively). The markup ratio and the intertemporal substitution parameter are set at the benchmark values of 1.6 and 2.5, respectively. Consider column (a), which assumes that the current patent length of 17 years represents the existing IPR protection level. Row 1 indicates that the corresponding $\mu_0 = 0.032$ is way above the optimal μ^*, which means that IPR is currently under-protected. Rows 2 to 4 report what would happen if optimal IPR protection were pursued. The steady state growth rate would accelerate to 2.67% (Row 2), comparing with the current growth rate of 1.6%; this is the dynamic gain from tightening IPR protection to the optimal level. There will be a static loss, however, due to a downward level shift of the consumption path. The extent of such consumption level shift, as reported in Row 3 for the normalized consumption variable h at the steady state, is 20.36%. The dynamic gain will of course outweigh the static loss and the welfare gain is 17.16% as reported in Row 4. Columns (b) and (c) assume weaker levels of current IPR protection, corresponding to CE monopoly durations of 5 and 10 years, respectively. As can be seen, the extent of the under-protection is greater, the weaker is the current IPR protection level. The reason is that, given the real interest rate r fixed at the observed value, a higher risk of imitation μ_0 implies that the underlying rate of return to innovation r_m must also be higher. The higher rate of return to innovation generates a wealth e®ect and an intertemporal substitution e®ect. The wealth e®ect motivates the benevolent social planner to take more current consumption and hence implies weaker optimal IPR protection. The intertemporal substitution e®ect, on the other hand, favors future consumption and hence implies stronger optimal IPR protection. Under our calibrated parameter values, the intertemporal substitution e®ect dominates the wealth e®ect.

Table 2 reports the results for a high markup ratio of 2.5, which corresponds to the value of ® determined by appealing to the conventional labor share value 1 ¡ ® = 0:6. Comparing with Table 1, it can be seen that a higher markup ratio implies slightly stronger optimal IPR protection. To check robustness, we also report in Table 3 the case for a low markup ratio of 1.25. Comparing the three tables, it can be seen that the results, especially the welfare implications, are rather robust with respect to di®erent markup ratios. The results in Tables 1 - 3 all suggest under-protection of IPR, and the welfare loss can be quite substantial. Will there ever be over-protection? If yes, is the welfare loss simply a mirror image of the under-protection case? How sensitive is our conclusion with respect to di®erent parameter values?

We have performed extensive numerical computation to investigate the issues raised above. As mentioned before, the result is insensitive to variations in the markup ratio 1=®. We thus concentrate on µ, the intertemporal substitution parameter, and $¹_0$, the current imitation rate, which parameterizes the current degree of IPR protection. Figure 8 depicts the log deviation of $¹_0$ from the optimal level $¹^¤$ over wide ranges of µ and T, the CE monopoly duration (in years) that corresponds to $¹_0$. The over-protection region is highlighted by triangles. As can be seen, over-protection happens only when the current IPR protection is already very strong, to the extent that a monopoly position is expected to last over 50 years. For most industries this is certainly not a plausible degree of current IPR protection. But it is interesting to note that the current US copyright protection is indeed 50 years, and that Coca-Cola has been successfully safeguarding its secret formula for 113 years! Figure 9 depicts the welfare gain from pursuing optimal IPR protection, for the same ranges of µ and T values as in Figure 8. It can be seen that the welfare loss is trivial for the over-protection region. In fact, for T over 35 years, the welfare loss from sub-optimal IPR protection has already become trivial, as indicated by the °at region in Figure 9. The reason is that when T is already very high (i.e., $¹_0$ very small), given the real interest rate ¯xed at the observed value, the zero pro¯t condition implies a low rate of return to innovation. There is thus not much growth e®ect to exploit by tightening IPR protection in the under-protection case. In the over-protection case, pursuing optimal IPR protection amounts to raising the level of the consumption path at the expense of the growth rate. As is well known from growth theory, the elasticity of welfare with respect to level e®ect is a lot smaller than that of growth e®ect. Therefore, the welfare gains from correcting over-protection of IPR is very small.

4 Concluding remarks

It must be borne in mind that our model is, in a sense, a metaphor that tries to capture the fact that development of new intermediate goods increases labor productivity, and that strengthening IPR protection tends to lengthen the duration of monopoly position of the innovators of these intermediate goods. Just like other one-sector, highly aggregated, macroeconomic models, our model cannot capture things such as sectoral di®erences. However, we gain in tractability, which allows us to calibrate the model using easily available macroeconomic data.

We have made use of a very simple model to illustrate the tradeo® between static losses and dynamic gains of IPR protection. While we gain from simplicity of the model, we might not have su±ciently captured certain important aspects of the economy. For example, the expanding variety model is subject to the criticism that it fails to capture obsolescence of goods | goods stay in the market forever. The absence of obsolescence might lead to an over-estimation of the degree of IPR under-protection that we have found to prevail in the real world. However, it is straightforward to show that the qualitative aspects of the results would be preserved if an exogenous rate of obsolescence is incorporated in the present model. It is endogenous obsolescence that is of substantive interest and has not been captured in this paper. We have extended the current model to incorporate endogenous obsolescence along the line of Lai (1998b) and the result will be reported elsewhere.

Recently, there has been debate on how to reconcile the historical trend of increasing R&D to output ratio and constant growth rate (see for example, Jones 1995 and Kortum 1997). Our model features constant R&D to output ratio, which is at odds with the fact. However, we believe our results will not be altered qualitatively even if we adopted a modi¯cation such as in Jones (1995), which features increasing R&D to output ratio.[3]

[3] However, if we did that, long-term growth would not be driven by R&D, which would be inconsistent with our original premise that IPR protection should have long-term growth e®ect.

Another extension we are carrying out is to assume a more general imitation technology to capture the fact that the rate of imitation is dependent also on the knowledge accumulated from past imitations. In this way, we allow for transitional dynamics of the hazard rate when there is a shock to IPR protection. Preliminary results show that this would give rise to multiple steady states and substantially richer transitional dynamics. It would be interesting to examine the qualitative and quantitative impact of changes in IPR protection in such a more general and presumably more realistic model.

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Table 1 (average markup)

$r = 0.065;\ \gamma_c = 0.016;\ 1/\alpha = 1.6;\ \theta = 2.5$	(a)	(b)	(c)
Current IPR protection level η_0	0.032	0.071	0.169
(CE monopoly duration in years corresp. to η_0)	(17)	(10)	(5)
1. Deviation from optimal IPR protection: $\ln(\eta_0/\eta^*)$	1.78	1.97	2.02
2. Optimal growth rate %	2.67	4.04	7.47
3. Consumption level shift %	-20.36	-26.04	-29.85
4. Welfare gain %	17.16	33.65	55.57

Table 2 (high markup)

$r = 0.065;\ \gamma_c = 0.016;\ 1/\alpha = 2.5;\ \theta = 2.5$	(a)	(b)	(c)
Current IPR protection level η_0	0.032	0.071	0.169
(CE monopoly duration in years corresp. to η_0)	(17)	(10)	(5)
1. Deviation from optimal IPR protection: $\ln(\eta_0/\eta^*)$	1.87	2.06	2.12
2. Optimal growth rate %	2.69	4.08	7.56
3. Consumption level shift %	-18.22	-23.10	-26.38
4. Welfare gain %	18.63	35.43	57.19

Table 3 (low markup)

$r = 0.065;\ \gamma_c = 0.016;\ 1/\alpha = 1.25;\ \theta = 2.5$	(a)	(b)	(c)
Current IPR protection level η_0	0.032	0.071	0.169
(CE monopoly duration in years corresp. to η_0)	(17)	(10)	(5)
1. Deviation from optimal IPR protection: $\ln(\eta_0/\eta^*)$	1.73	1.92	1.97
2. Optimal growth rate %	2.66	4.02	7.43
3. Consumption level shift %	-21.30	-27.41	-31.54
4. Welfare gain %	16.32	32.62	54.63

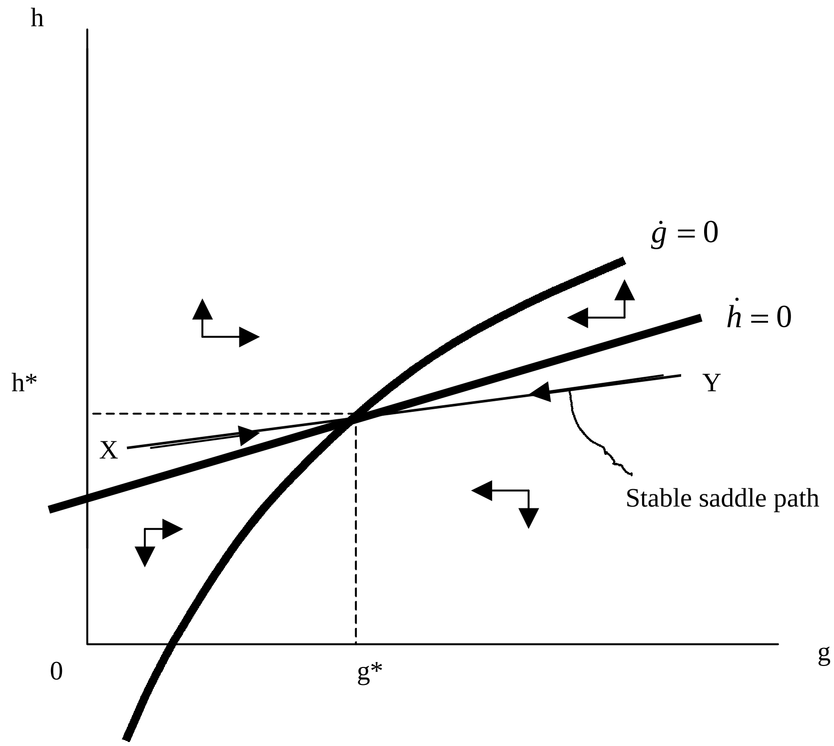


Figure 1. Transitional dynamics

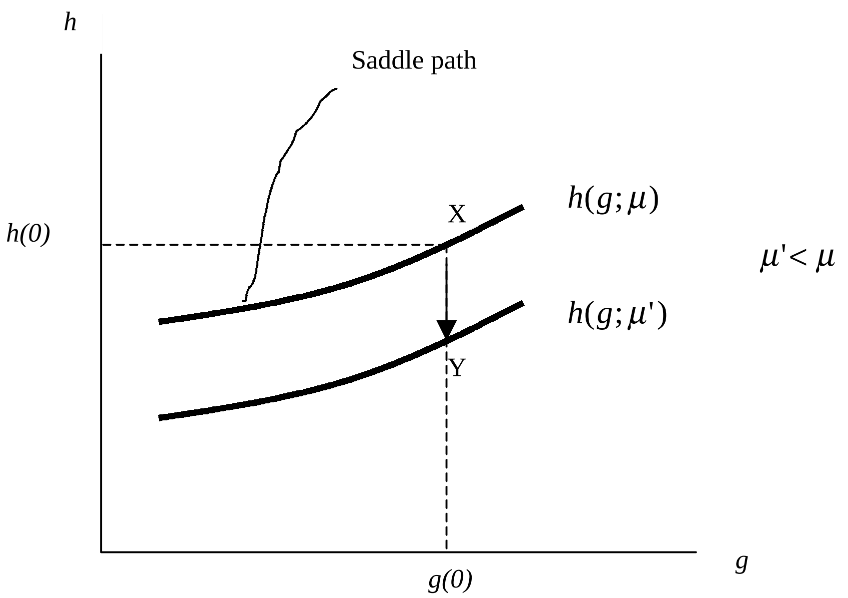


Figure 2. The shift of the saddle path as μ changes

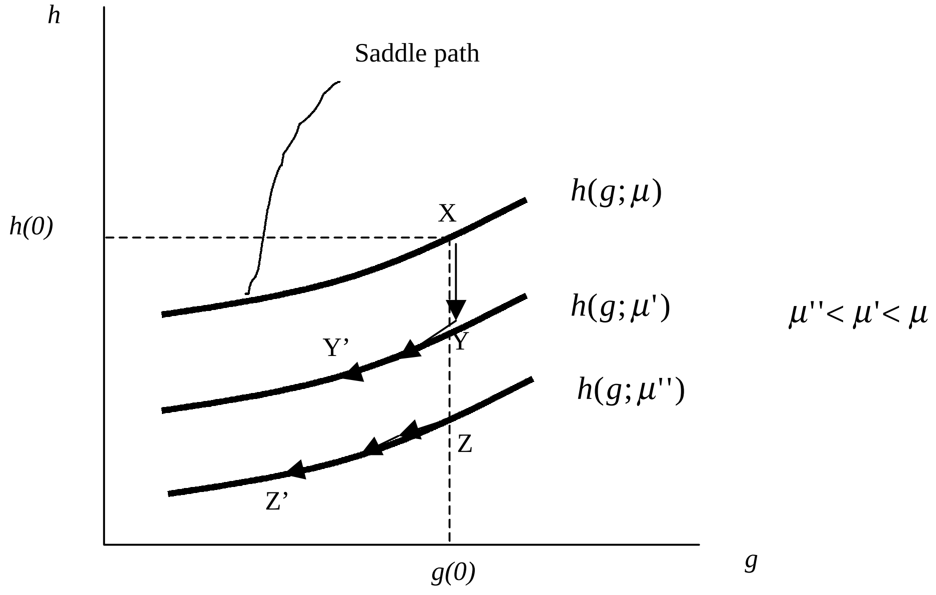


Figure 3. Transitional dynamics of *g* and *h* when there is a shock in μ.

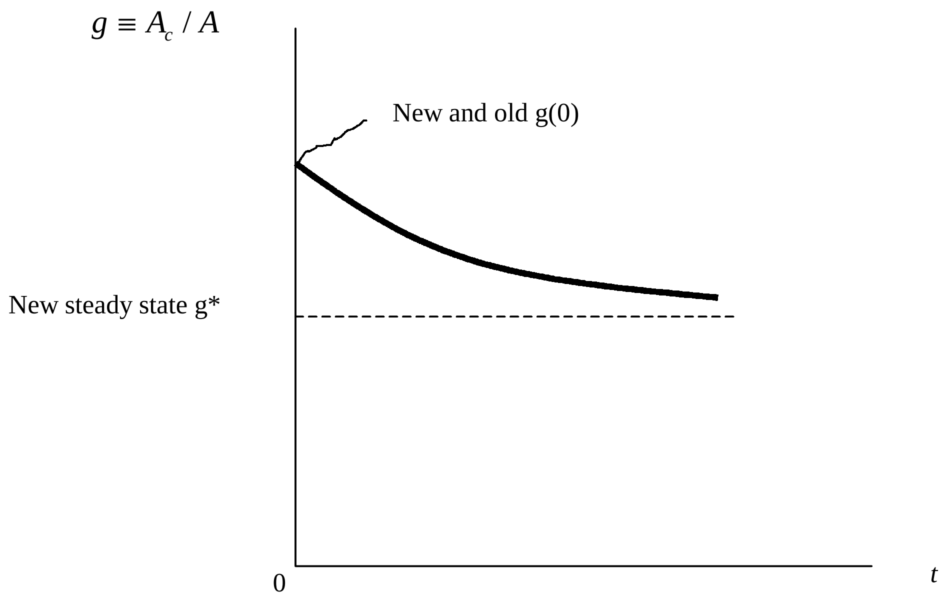


Figure 4. The time path of g with a tightening of IPR protection at t = 0

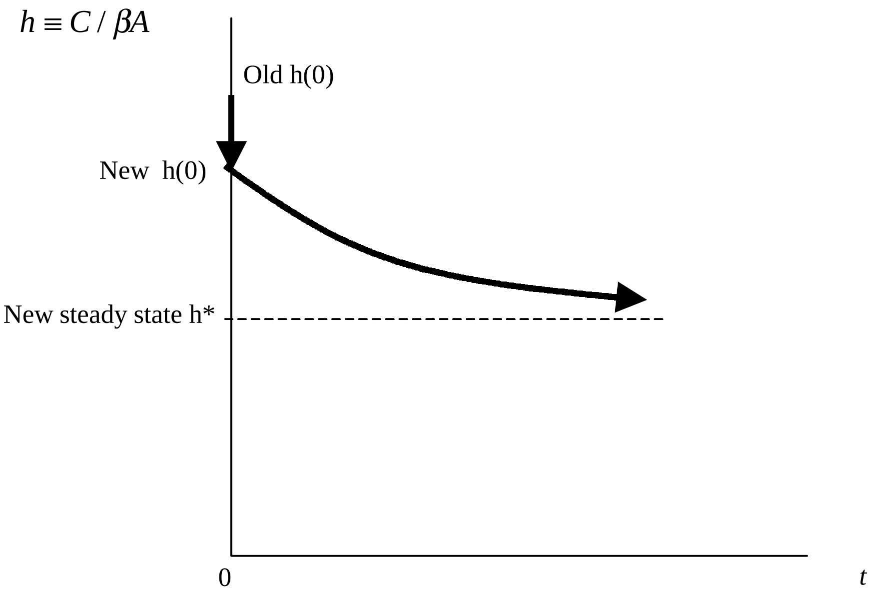


Figure 5. The time path of *h* with a tightening of IPR protection at t = 0

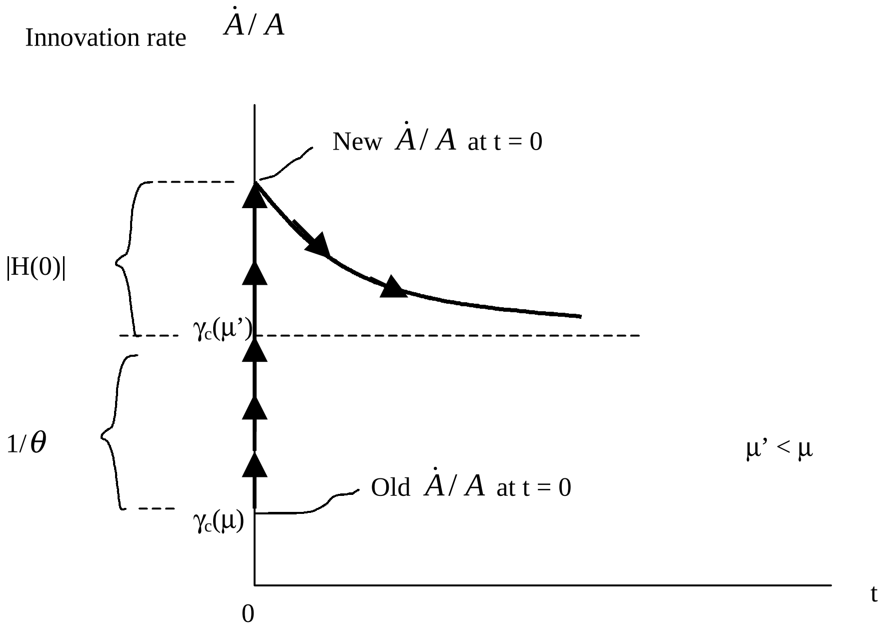


Figure 6. The impact of tightening IPR protection (lower μ) on the innovation rate

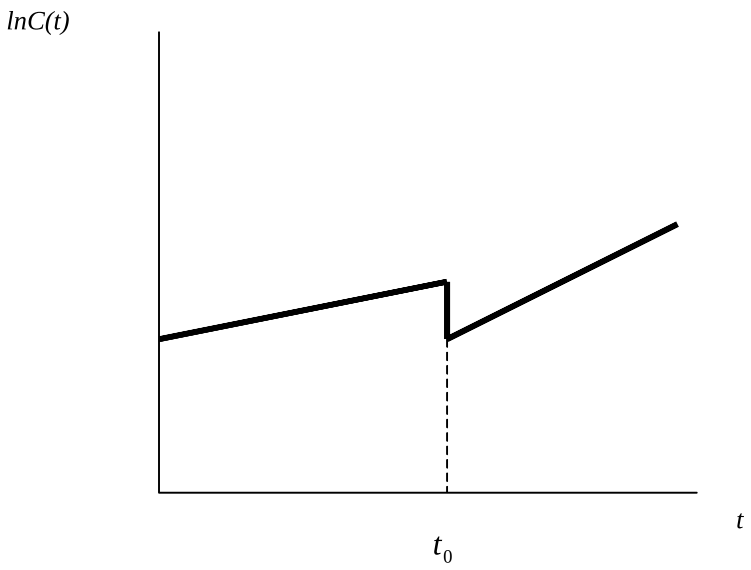
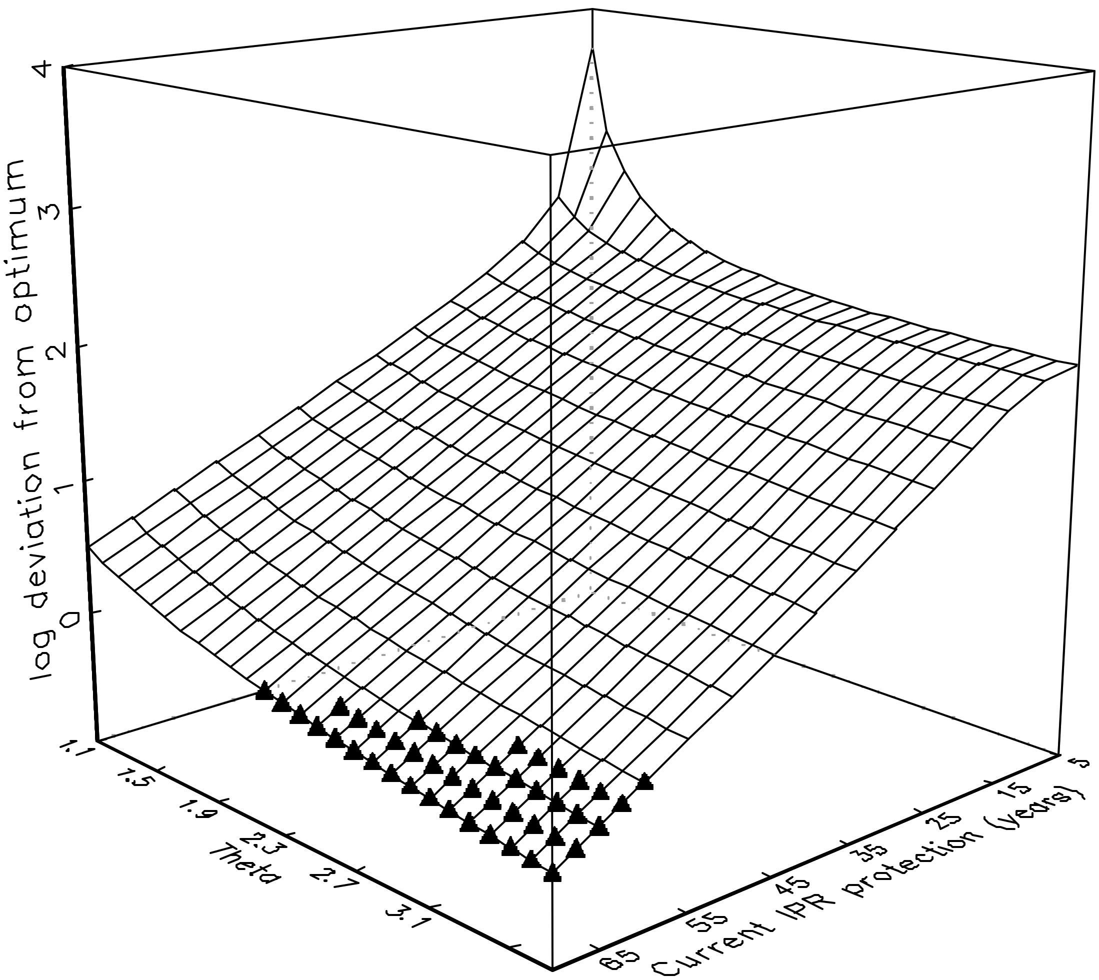


Figure 7. The time path of *C(t)* when a tightening of IPR protection is announced at date t_0

Imitation rate μ



Welfare gain