Increasing wealth and increasing instability*

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Abstract

When will financial crisis happen? It is by now well known that an important precondition is high growth in domestic credit. However, growth in credit is generally associated with faster long run growth by authors writing in finance and development. Both the empirical and theoretical studies in this literature point to the conclusion that higher level of financial intermediation improves the efficiency of channelling capital to productive investment and improving growth as a result. This paper builds an endogenous growth model to take a preliminary step in reconciling these strands of investigations. If there is a significant fixed cost for joining intermediation, households are excluded from the credit market at low levels of development, thereby insulated from the disturbance of the financial market. When agents accumulate enough capital to cross the threshold, they take the advantage of international borrowing, but becomes leveraged and vulnerable to the shocks in world capital market.

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1 Introduction

It is now widely accepted that well-functioning financial markets are growth enhancing.[1] Among others, King and Levine (1993) have established a strong link between financial development and growth in a large cross-section of countries.[2] This, however, begs the question of why do not all developing countries choose to have a liberalized financial market in the first place. Greenwood and Jovanovic (1991, GJ hereafter) point out that participation in financial intermediation entails significant fixed costs for individual agents. It is only when sufficient wealth has been accumulated that it pays to join financial intermediation. The development of the financial market, too, should be viewed as an endogenous process. This paper focus on one of the channels that financial markets will promote growth, which is the allowance for domestic firms to borrow from the international capital market. The growth enhancing role of foreign capital inflow has also long been noted by development economists. For instance, unrestricted international borrowing enables a country to attain convergence to its steady state output level instantaneously in the neoclassical growth model.[3] This paper extends the intuition of GJ to international borrowing and suggests that countries would only choose to borrow in the international capital market after it has reached a certain stage in its development process. Once they have access to the international capital market, they would grow faster.

However, recent events in East Asia (and earlier ones in Latin America) indicate very clearly that international capital flows could also have devastating effects on the borrowing economies.[4] This should not be surprising from the lessons learned in the closed economy model of leverage borrowing introduced by Kiyotaki and Moore (1997, KM hereafter).[5] They show that when households engage in maximum leverage borrowing, tiny productivity shocks can result in huge fluctuations in output through the induced fluctuations in the values of collateral. Again, extending the intuition to countries engaging in leverage international borrowing, it is none too surprising that they too should be extremely vulnerable to shocks in the world capital market.

The combined picture painted by the two strands of literature is that countries in the early stage of development would choose not to borrow in international capital market even if the world interest rate is below the domestic user cost of capital. They are largely insulted from shocks in the world capital market but grow slowly. Entry to the world capital market takes place when the country has accumulated sufficient wealth. International leverage borrowing enables the country to achieve faster growth thereafter while in the meantime, it becomes vulnerable to shocks in the world capital market.

In this paper, we construct a canonical model of a small open economy to formalize the above idea as a first step to understand the complicated interaction between international borrowing, development and fluctuations. The basic structure is from the Ak endogenous growth model of Rebelo (1991).[6] We add the possibility of international borrowing at an interest rate below the domestic user cost of capital. Much in the same way as in KM, we assume that the representative household in the domestic economy may only borrow up the value of the collateral, which we take as the stock of physical capital. This imposes an upper bound to borrowing which would otherwise be absent in a model with constant returns to scale.[7]

Assuming that there is a one-time fixed cost to be incurred for entry to international borrowing, the poorest countries would choose not to engage in international borrowing, just as in GJ where agents would choose to wait until sufficient wealth has been accumulated before joining financial intermediation. GJ model access to financial intermediation as gaining access to a better investment technology that yields a higher return to investment. Ours has a more detailed microeconomic foundation in that we model access to financial intermediation as the access to borrowing and lending at the world interest rate.

By construction, the poorest countries in the model would not have borrowed internationally. They grow slowly but they are insulated from shocks in the world capital market. On the other hand, we show that the magnitude of the fluctuations for the faster growing countries that have borrowed internationally could be huge.

The next section presents the model of international leverage borrowing and endogenous entry to the world capital market. Section 3 analyzes the response of a borrowing economy to shocks to the world interest rate and international credit ratings. Section 4 concludes.

2 Model

2.1 Basics

We study a model of a small open economy. There are two types of goods : output and physical capital. There is a world market for output and for physical capital, whose prices are both equal 1. The economy is populated by a continuum of identical households with total measure equal to 1. Time is continuous and the horizon is infinite. The discount factor is ρ. Discounted lifetime utility of the representative household at time 0 is

$$U = \int_0^\infty e^{-\rho\tau} \ln c(\tau)\, d\tau$$

where $c(\tau)$ is consumption at time τ. Let $k(\tau)$ be capital input. The household-producer has assess to a Ak technology with output given by

$$y(\tau) = Ak(\tau). \tag{1}$$

At time 0, the household has an endowment of capital $k(0) = k_0$. Let δ be the rate at which capital depreciates and $x(\tau)$ investment in physical capital. The household's holding of physical capital evolves according to

$$\dot{k}(\tau) = x(\tau) - \delta k(\tau). \tag{2}$$

Throughout the analysis that follows, we assume that:

(A1) $A > \rho + \delta$.

The assumption is a standard one that allows for positive endogenous growth for a closed economy with Ak technology and logarithmic preference.

2.2 Finance and Development

There is world capital market with a fixed interest rate r. However, access to this market is not unrestricted. In practice, the market for lending and borrowing, especially one that crosses national borders, is plagued by a whole array of information problems that makes it difficult for individuals to locate suitable trading partners, and for creditors to monitor their investment. Financial intermediaries arise to help match lenders and borrowers and monitor investment on behalf of their clients, among the other functions they perform. But these services are not available for free. More important is the observation that domestic firms may have to alter their accounting conventions and other practices to convince foreign as well as domestic financial intermediaries that they represent prudent investment. We model the resources that the domestic household-producer would have to spend to gain access to the world capital market as the payment of a one time fixed cost equal to θ units of capital.[8]

With a Ak technology, the net marginal product of capital is constant at $A-\delta$. If the world interest rate $r > A - \delta$, the household is better off shutting down domestic production and invests all resources abroad. Conversely, if $r < A - \delta$, domestic production yields higher returns than investment abroad. Since our analysis is about growth and instability from international borrowing for an indebted developing country, the latter case is more relevant for our purpose.

(A2) $A > r + \delta$.

Under the above assumption, the household would like to borrow the maximum possible by virtue of its access to a constant returns technology. As in KM, we assume that the maximum the household can borrow is constrained by the value of collateral that the household may be able to put up.

Nevertheless, the household may not want to enter the world capital market to borrow at time 0 even if the user cost of capital is below the marginal product. At low level of wealth, the sacrifice it may have to make to join intermediation (the payment of θ) will outweigh the benefit. It may be optimum for the household to wait for a finite period of time to accumulate enough wealth before entering the world capital market to engage in leverage borrowing.

In the next section, we will describe the household's problem and the equilibrium assuming that it has already sunk θ to gain access to the world capital market. Then we will backtrack to describe the household's problem and the equilibrium before doing so.

2.3 Equilibrium with international borrowing

Let T be the time the household joins international financial intermediation. Let's say its capital holding at T after paying the entry cost θ is equal to some $k(T) = k_T$. Denote $b(\tau)$ as its borrowing at $\tau \geq T$. The amount of debt at any time is constrained by the values of the household's capital stock to be used as collateral for borrowing[9] :

$$(1+r)\, b(\tau) \leq (1-\delta)\, k(\tau)\,. \tag{3}$$

The borrowing constraint will be binding if (A2) holds :

$$b(\tau) = m_b k(\tau)\,, \tag{4}$$

where

$$m_b \equiv \left[(1-\delta)/(1+r)\right],\ 0 < m_b < 1.$$

International borrowing allows the household to amass more resources than what it actually owns. Let $e(\tau)$ be the equity of the household in the firm that it owns. We have the identity:

$$e(\tau) = k(\tau) - b(\tau)\,.$$

Applying (4)

$$e(\tau) = k(\tau)/m_r, \tag{5}$$

where

$$m_r \equiv (1+r)/(r+\delta) > 1.$$

The above captures the notion of leverage borrowing in that the household's equity is only a fraction of the asset of the firm.

At time T, the instant the household enters the world capital market, it does not yet owe any debt. Indeed, it owns all the capital stock that the firm possesses then : $e(T) = k(T)$. Furthermore, the path for e must be smooth throughout as the household cannot accumulate any resources on its own in an interval of zero length.[10] This implies $e^{+}(T) = e(T)$ where the former denotes equity at an instant after T. In all, the equity–asset relationship in (5) at the instant after T becomes

$$k(T) = k^{+}(T)/m_r.$$

The above implies that there will be a discrete jump in k the moment after entry to the world capital market. In other words, international borrowing raises the effective initial capital stock that the household possesses : with an initial equity $e(T) = k(T)$, the household would be able to bring in enough resources through borrowing to jack up its holding of capital to:

$$k^{+}(T) = m_r e(T) = m_r k(T).$$

It should be emphasised that the above analysis comes entirely from the equity equal asset minus debt identity rather than from any maximizing behaviors of the household at T. Accordingly, it should hold so long as the household-producer chooses to engage in maximum leverage borrowing.

At any time $\tau > T$, the receipts of the household consist of its output from production and any increase in borrowing. Its expenditure includes consumption and investment spending, as well as interest payment. This yields the budget constraint

$$y(\tau) + \dot{b}(\tau) = c(\tau) + x(\tau) + rb(\tau). \tag{6}$$

Using (4), (2), and (1), the budget constraint becomes[11]

$$\dot{k}(\tau) = m_r (A - \delta - m_b r) k(\tau) - m_r c(\tau) \quad \tau > T. \tag{7}$$

The household's problem is

$$V_I(k_T) = \max \left\{ \int_T^{\infty} e^{-(\tau - T)\rho} \ln c(\tau) \, d\tau \right\} \tag{8}$$

subject to (7) and the initial condition :

$$k^{+}(T) = m_r k_T. \tag{9}$$

The Hamiltonian is

$$\mathcal{H}(\tau) = e^{-(\tau-T)\rho} \ln c(\tau) + \lambda(\tau) \{ m_r (A - \delta - m_b r) k(\tau) - m_r c(\tau) \}. \quad \tau > T,$$

where $\lambda(\tau)$ is the Lagrangian multiplier associated with the constraint in (7). Solving the optimum control problem yields a Euler equation

$$\dot{c}(\tau) / c(\tau) = m_r A - 1 - \rho. \tag{10}$$

There is also a transversality condition:

$$\lim_{\tau \to \infty} \lambda(\tau) k(\tau) = 0. \tag{11}$$

We do not impose a no-pronzi game condition on borrowing because we have restricted the household's debt to be perfectly collateralized.

Eq. (10) implies that consumption grows at a constant rate in equilibrium, just as in standard Ak endogenous growth model. Call this constant growth rate γ_c. It is straightforward to verify that γ_c exceeds $A - \delta - \rho$, the growth of consumption in standard Ak model with logarithmic preference. And by (A1), positive consumption growth is assured. Constant consumption growth allows rewriting (7) as

$$\dot{k}(\tau) = m_r (A - \delta - m_b r) k(\tau) - m_r c^+(T) e^{\gamma_c(\tau - T)} \tag{12}$$

where $c^+(T)$ is consumption the instant just after T. Remember that the above is only valid for $\tau > T$. Had there been no jump in the state variable k at T, $c(\tau)$ would be smooth throughout and $c^+(T) = c(T)$. With the jump, c may be discontinuous at T and that makes it necessary to distinguish $c^+(T)$ from $c(T)$.

Eq (12) is a linear differential equation in k. The general solution of (12) takes the form :

$$k(\tau) = (m_r/\rho) c^+(T) e^{\gamma_c(\tau - T)} + Be^{\Phi(\tau - T)} \tag{13}$$

where B is the constant of integration and $\Phi = m_r (A - \delta - m_b r)$. It can be shown that the Lagrangian multiplier $\lambda = e^{-\rho(\tau - T)} m_r/c$ for $\tau > T$. The transversality condition in (11) then reduces to the requirement that

$$e^{-\rho(\tau - T)} (m_r)^{-1} (k/c) = \rho^{-1} e^{-\rho(\tau - T)} + (m_r)^{-1} B/c^+(T) \tag{14}$$

asymptotes to 0. The first term above obviously converges to 0. Hence the transversality condition calls for a $B = 0$. Then from (13),

$$k(\tau) = (m_r/\rho) c(\tau).$$

We are now in the position to calculate the discounted lifetime utility at time T of the household whose capital holding then is equal to some k_T. This calls for pinning down the consumption path. We know that $c(\tau)$ is smooth throughout with the possible exception at T. In particular, for $\tau > T$

$$\begin{aligned} c(\tau) &= (m_r)^{-1} \rho k(\tau) \\ &= (m_r)^{-1} \rho k^+(T) e^{\gamma_c(\tau - T)} \\ &= \rho k(T) e^{\gamma_c(\tau - T)}. \end{aligned} \tag{15}$$

We have yet to determine consumption at T, $c(T)$. This is not necessary, however, because its value will have zero weight in discounted lifetime utility.[12] The knowledge of $c(\tau)$ for $\tau > T$ suffices for calculating discounted lifetime utility. Finally, evaluating (8) using (15) gives

$$V_I(k_T) = \rho^{-1} (\ln k_T + \ln \rho + (\gamma_c/\rho)). \tag{16}$$

To summarize, beginning with $k(T) = k_T$, there will be a discreet jump in k just after T. After the jump, the consumption–capital ratio is $(m_r)^{-1} \rho < \rho$, which is the consumption–capital ratio without any international borrowing. This means that the optimum accumulation path with borrowing results in lower consumption as a fraction of capital stock. But the fall is entirely compensated by the jump in the capital stock right after T. And with $A > r + \delta$, the household is onto a faster growth path.

It is of interest to note that with constant growth, at each $\tau > T$, the net amount of resources made available from international borrowing

$$\begin{aligned} \dot{b} - rb &= b\left(\dot{b}/b - r\right) \\ &= b\left(\gamma_c - r\right) \end{aligned}$$

is either positive or negative throughout. If $\gamma_c < r$, the household relinquishes resources to the outside world at each $\tau > T$. It is nevertheless better off if $A - \delta > r$ with international because the outflow of resources is more than compensated by the inflow at $\tau = T$. The current account is in deficit at T but is in surplus thereafter. On the other hand, if $\gamma_c > r$, there is net inflow of resources into the economy at T as well as at each moment thereafter. The current account is perpetually in the red.

Since the amount of debt grows at γ_c, the present value grows at $\gamma_c - r$. In case $\gamma_c < r$, the present value of debt asymptotes to 0. This is usually thought to be the condition necessary for current account sustainability.[13] On the other hand, if $\gamma_c > r$, the present value of debt grows without bounds and the usual no-pronzi game condition is violated. But with the debt perfectly collateralized by asset that grows at the same rate, the household can not be considered as running a pronzi scheme.

2.4 Autarkic equilibrium

Now we backtrack to the problem the household faces before it has paid θ to gain access to international borrowing. Given the initial holding of capital : $k(0) = k_0$, it chooses investment at each τ and the timing of entry to the world capital market to maximize lifetime discounted utility.

$$V(k_0) = \max\left\{\int_0^T e^{-\rho\tau} \ln c(\tau)\, d\tau + e^{-\rho T} V_I\left(k(T) - \theta\right)\right\}.$$

The maximization is over $x(\tau)$ for $\tau \leq T$ and the timing of entry T, subject to a budget constraint that neither allows borrowing nor lending :

$$c(\tau) = Ak(\tau) - x(\tau). \tag{17}$$

and the equations of motion for $k(\tau)$ in (2). Notice that in the above, the investment decisions after T are subsumed in $V_I(.)$.

It is easiest to proceed by breaking up the problem into two parts. First define[14]

$$V_a(k_0, k^*, T) = \max_{x(\tau)}\left\{\int_0^T e^{-\rho\tau} \ln c(\tau)\, d\tau\right\} \tag{18}$$

subject to the initial condition, the budget constraint, and the equation of motion as specified above, as well as a terminal condition, $k(T) = k^*$. This gives the payoff before gaining access to international borrowing where the date of entry and the amount of capital to carry over are fixed respectively at some T and k^*. And the second step will be choosing T and k^* optimally.

Now the first step. The necessary conditions for maximizing (18) are :

$$\dot{c}(\tau)/c(\tau) = A - \delta - \rho. \tag{19}$$

Constant consumption growth allows rewriting the budget constraint in (17) as

$$\dot{k}(\tau) = (A - \delta) k(\tau) - c(0) e^{\gamma_c^a}$$

where γ_c^a is the growth rate of consumption as defined in (19). Solving this differential equation yields

$$k(\tau) = (c(0)/\rho) e^{(A-\delta-\rho)\tau} + Be^{(A-\delta)\tau} \tag{20}$$

where B is the constant of integration. The two constants in the above $c(0)$ and B can be fixed with the help of the initial and boundary conditions : $k(0) = k_0$ and $k(T) = k^*$:

$$c(0) = \rho \frac{k_0 - e^{-(A-\delta)T} k^*}{1 - e^{-\rho T}}, \tag{21}$$

$$B = \frac{e^{-(A-\delta)T} k^* - e^{-\rho T} k_0}{1 - e^{-\rho T}}. \tag{22}$$

Finally, evaluate the discounted utility from time 0 to T in (18) using $c(\tau) = c(0) e^{\gamma_c^a}$

$$V_a(k_0, k^*, T) = \rho^{-1} \left\{ \left(1 - e^{-\rho T}\right) [\ln c(0) + (\gamma_c^a/\rho)] - \gamma_c^a e^{-\rho T} T \right\}. \tag{23}$$

2.5 Entry to international capital market

We now have all the ingredients necessary to solve for the optimum entry date and the capital stock to prepare for the transition, T and k^*. To this end, add (23) to the discounted value of (16) to obtain discounted lifetime utility at time 0 :

$$V(k_0) = \max_{T,\ k^*} \left\{ V_a(k_0, k^*, T) + e^{-\rho T} V_I(k^* - \theta) \right\}.$$

Remember that the first term denotes discounted utility of the household from time 0 up to some fixed T with a fixed terminal capital stock $k(T) = k^*$. At time T, it enters the world capital market by paying the entry cost θ. The second term denotes the discounted utility from T onwards.

The necessary condition for optimum k^* is

$$-\left(\frac{1-e^{-\rho T}}{e^{(A-\delta)T}k_0-k^*}\right)+\frac{e^{-\rho T}}{k^*-\theta}=0.$$

Differentiating the left side further confirms that the second order condition is met for each $T\geq 0$. Hence the above is also sufficient for optimum at given T. Solving for k^*

$$k^*=e^{(A-\delta-\rho)T}k_0+\left(1-e^{-\rho T}\right)\theta. \tag{24}$$

The necessary condition for optimum T is

$$\rho e^{-\rho T}\left\{\ln c(0)+\left(\gamma_c^a/\rho\right)\right\}+\left(1-e^{-\rho T}\right)(c(0))^{-1}\left(\partial c(0)/\partial T\right)+\rho\gamma_c^a e^{-\rho T}T-\gamma_c^a e^{-\rho T}-$$
$$\rho e^{-\rho T}\left\{\ln\left(k'-\theta\right)+\ln\rho+\left(\gamma_c/\rho\right)\right\}\leq 0$$

with equality if $T>0$. Substitute (24) into the above and simplify

$$\frac{k_0e^{(A-\delta)T}+\left(e^{\rho T}-1\right)\theta}{k_0e^{(A-\delta)T}-\theta}\leq\frac{(1+r)A-r-\delta}{(r+\delta)(A-\delta)}. \tag{25}$$

Assuming $\rho\leq 1$, the left side is decreasing in T if $k_0e^{(A-\delta)T}>\theta$. Furthermore, note that $k_0e^{(A-\delta)T}$ is the maximum amount of physical capital that can be accumulated starting from $k(0)=k_0$ in an interval of length T. Hence, optimum T must be such that $k_0e^{(A-\delta)T}>\theta$; otherwise, there will simply not be enough to pay for the entry cost. Together, they imply that (25) is also sufficient for optimum for $T\geq(A-\delta)^{-1}\ln(\theta/k_0)$.

In a closed economy with Ak technology and logarithmic preference, $r=A-\delta$. At this value of r, it can be checked that the right side of (25) is equal 1. But the left side is greater than 1 for any positive θ except when $T\to\infty$. When the autarkic and world interest rate coincide, there is no benefit to entering the world capital market. Therefore it is never optimum to pay a fixed entry cost to do so.

If (A1) is satisfied, i.e. $r<A-\delta$, the right side of the inequality is greater than 1. At $\theta=0$, the left side is equal 1 at any $T\geq 0$. Obviously, at zero entry cost, it is optimum to enter the world capital market immediately.

In general, starting at $T=(A-\delta)^{-1}\ln(\theta/k_0)$, the left side of (25) declines from infinity towards 1 as T increases. Since the right side is greater than 1, there is always a finite optimum T. Eventual entry into the world capital market is optimum for any initial level of wealth k_0. Also it can be verified that for

$$k_0>\theta\frac{(1+r)A-r-\delta}{(1-\delta)(A-r-\delta)}\equiv k^s \tag{26}$$

the solution for T from (25) as an equality is negative. This means that there is a minimum level of k_0 equal to k^s as defined above at which immediate entry is optimum. Conversely, if the household starts with initial wealth below k^s, it will choose not to enter the world capital market immediately.

Remember that the growth rate of consumption before gaining access to international borrowing is some $A-\delta-\rho$. If not for the prospect of entering the world capital market, this will also be the growth rate of output and capital, as in a standard Ak model with logarithmic preference. The post-entry growth rate is at a higher level equal to $m_r A-1-\rho$. It is useful to think of the possibility of borrowing at a world interest rate below the autarkic one as the availability of a more productive technology, and the entry cost as the cost of adopting the more productive technology.[15] Under this interpretation, the above may be understood as the household not wanting to adopt a more productive technology at once because the fixed adoption cost introduces a non-convexity so that the use of the better technology may only be justified at high level of input.

Note that from (26), $k^d > \theta$. This means that the threshold level of wealth that would induce the household to enter the world capital market is greater than the minimum necessary. The household would not choose to enter once after it is technically feasible to do so. It prefers to enter only after passing this necessary minimum by an sufficient amount.

3 Response to shocks

In the last section, we present a canonical model of endogenous growth with international borrowing that shows faster steady state growth will result if the country has access to international borrowing at a sufficiently low interest rate. Our next task is to check the response of the economy to external shocks pertaining to conditions in the world capital market.

In particular, consider an economy that has been engaging in maximum leverage borrowing. At some time t, the world interest rate rises to r' unexpectedly for an interval of Δt. We ask what would happen to the paths for output, capital and consumption?

The answer depends crucially on the terms of the country's external debt. It is well known that most developing countries' debt are short term debt.[16] Furthermore, the mircoeconomics underlying the borrowing constraint in (3), as explained in KM, calls for debt with the shortest possible term – one period in the discrete time model of theirs.

Incidentally, the effects of unexpected changes in conditions in the world capital market on the domestic economy are most pronounced when the debt is of shortest duration. Unexpected temporary increases in world interest rate would not have much of an effect on the cost of borrowing if most existing debt can be continuously serviced at the pre-shock interest rate. It is only the addition to existing borrowing which may be dearer to service if most of the country's external debt is long term debt. Conversely, if the debt is of a term $dt \simeq 0$ so that all debt matures in the next instant, all borrowing must be serviced at the higher post-shock interest rate. More important is the effect on the amount that may be borrowed. By virtue of (5), the capital that the household may amass as a function of its equity is.

$$k(\tau) = m_r e(\tau). \tag{27}$$

The equity of the household is the fraction of the capital stock that it owns. An unexpected increase in the interest rate leaves it unchanged but lowers the multiplier m_r. If the existing debt is yet to mature, the capital stock that the household may hold would not have to be adjusted even if the equity of the household may only strictly permit it to borrow a smaller amount at the higher interest rate. On the other hand, with existing debt maturing at every instant, the relationship in (27) would hold at all time. In particular, a given increase in interest rate from r to r' at t means that at the instant just after t, the capital stock the household has access to would experience a discrete downward adjustment to

$$k^+(t) = m_r' e(t),$$

where $m_r' \equiv (1+r')/(\delta+r')$, from the initial level at $k^-(t) = m_r e(t)$ when all existing debt are recalled and only a discretely smaller quantity of new loan would be extended at the new interest rate. With maximum leverage borrowing, the adjustment would be dramatic. Specifically,

$$\left| \frac{r}{k}\frac{dk}{dr} \right| = \frac{r}{r+\delta}\frac{1-\delta}{1+r}.$$

Under usual parameterization, the above is only slightly below 1.

It is possible to undertake a more exact analysis of how the economy responds to the positive interest rate shock. At time t, the instant after the increase in interest rate, there will be a discrete downward adjustment of capital stock as described above. This pins down the capital stock that remains at the instant just after t. If the shock is expected to last for an interval of Δt, the household's optimal response is the solution to

$$\max_{x(\tau)} \left\{ \int_t^{t+\Delta t} e^{-\rho(\tau-t)} \ln c(\tau)\, d\tau + e^{-\rho\Delta t} V_I \left(k^+(t+\Delta t) \right) \right\} \tag{28}$$

subject to the state equation for k in (7) with r replaced by r' and

$$k^+(t+\Delta t) = (m_r/m_r')\, k^-(t+\Delta t) \tag{29}$$

and $k^+(t)$ given.

To understand (29), first recall that the interest rate is expected to return to the original level r at $t+\Delta t$. Just before so,

$$e^-(t+\Delta t) = m_r' k^-(t+\Delta t).$$

The condition in (29) follows once we recognize that the path for e must be smooth. In the above, we assume that the post-shock interest rate r' is still below the user cost of capital $A-\delta$. Of course, this needs not be true. When the world interest rate rises above the $A-\delta$, all international borrowing would come to a halt. We modify the above by setting $(m_r')^{-1}=1$ and use the state equation implied by (17) instead of (7).[17]

We may solve this problem by breaking it into two parts, much like the procedure we employ previously in solving for the optimal entry to international borrowing. In the first step, we solve for the optimal consumption path in the interval $(t,t+\Delta t)$ taking as given a fixed terminal capital stock $k^-(t+\Delta t)$. It is then possible to calculate the sum of discounted utility for the interval $(t,t+\Delta t)$ given the fixed terminal capital stock. Together with the known closed form for V_I, this enables us to arrive at a closed form expression for discounted lifetime utility at t as a function of $k^-(t+\Delta t)$. Finally, discounted lifetime utility is maximized with respect to $k^-(t+\Delta t)$.

Following the procedure just sketched, it can be shown that[18] during the episode of unexpectedly high interest rate, the growth rates of consumption, capital and output are all constant at $\gamma_c = m_r'A-1-\rho$.[19] Since without the shock, the growth rate would have been $\gamma_c = m_rA-1-\rho$, the change in growth induced by the shock is simply

$$\frac{\partial\gamma_c}{\partial r} = \frac{\delta-1}{(r+\delta)^2}A.$$

To see the approximate magnitude of the above, assume that the world interest rate r is only slightly below $A-\delta$, then $\partial\gamma_c/\partial r \simeq (\delta-1)/(r+\delta)$. This could be an extremely large negative number under the usual values for r and δ.

As pointed out in note 11, we may write the borrowing constraint in the general form $b(\tau)\le\mu k(k)$ for any $\mu<1$. It can be shown that in this case, growth rate is

$$\gamma_c = m_\mu - \rho$$

where $m_\mu = (A-\delta-r\mu)/(1-\mu)$ and the capital-equity ratio $(k/e)=(1-\mu)^{-1}$. The parameter μ may be thought of as a function of credit ratings. Shocks to μ can lead to the following effects on the growth and level of output

$$\begin{aligned}\partial\gamma_c/\partial\mu &= m_\mu/(1-\mu),\\ \frac{\mu}{k}\frac{\partial k}{\partial\mu} &= \frac{\mu}{1-\mu}\end{aligned}$$

which can be large numbers even for fairly small μ.

All together, the economy could be extremely vulnerable to unexpected changes in the world interest rate and perhaps the credit ratings of the economy, in both level as reflected in the possible large drop in the level of capital just after shock and in growth during the interval of high interest rate and bad credit ratings.

Remember that international borrowing can be thought of as the availability of a better technology with net marginal product of capital m_rA-1 (m_μ in case when the borrowing constraint is of the general form) which is greater than $A-\delta$ the net marginal product of capital in a closed economy for $r < A-\delta$. The unexpected increase in r (or decline in μ) is analogous to a deterioration of the better technology. With logarithmic preference, we find that growth during the episode of low productivity (high interest rate or bad credit ratings), as well as in other times, is equal to $m_r A - 1 - \rho$ ($m_\mu - \rho$ in the general case) whatever r (μ) turns out to be[20].

This result is sensitive to the assumption of logarithmic preference. "Lower productivity" during the episode of high interest rate (bad credit ratings) does not lead to higher or lower rate of accumulation other than the adjustment called for by the change in "productivity" per se because intertemporal substitution and wealth effects exactly cancel out with logarithmic preference. If intertemporal substitution is sufficiently strong, the household may want to slow down accumulation even further.

In closing, we check how large the effects can be under the usual parameter values. Assume $\delta = .1$, $\rho = 0.02$, $r = 0.04$ and $A = 0.145$. The values for (δ, ρ, r) are at their usual values while A is chosen to calibrate a closed economy growth rate of $\gamma_c^a = 0.025$. These parameter values imply a pre-shock open economy growth rate of $\gamma_c = 0.057$. We take that the post-shock interest rate rises to a level that exceed the user cost of capital $A-\delta$ so that all international borrowing would come to a halt. This implies a drop in growth rate during the interval of high interest of $\gamma_c - \gamma_c^a = 0.032$. But there is also a discrete drop in capital (and output) at the instant of the shock from $k^-(t) = m_r e(t)$ to $k^+(t) = e(t)$. Under the parameter values we adopt, capital (and output) after the shock is only 13% of the level before the shock. This prediction, which is a result of assuming maximum leverage borrowing with an equity-capital ratio of $(m_r)^{-1} = 0.13$, is obviously too extreme to believe. But the point that the effect can be huge remains valid.

4 Concluding remarks

Economic crises are often catalysts for sophisticated economic research. For instance, the Great Depression in 1930s has inspired generations of economists to develop new theory. Keynes (1936), Friedman and Schwartz (1963), Lucas (1972), among others, are all intellectual responses to it and they generate waves of dramatic changes in macroeconomics. Unfortunately, while the profession has yet to come up with a widely accepted interpretation of the Great Depression, there is another just as great challenge to take up. It is the Asian Crisis. When economists, and so as the general public, is debating whether the postwar economic performance of East Asia is just a myth or a true miracle, a sequence of seemingly small events triggers a sequence of collapses in a couple of Asian countries. And even the most optimistic analysts will agree that the Crisis is yet to be over. Many papers has been written on this subject and this paper is one of them. This paper focuses on the "timing" of the event. In particular, why would the crisis happen now but not any time before? It seems that a static coordination failure story cannot provide a satisfactory answer to that. In fact, empirical work suggests that there are very clear signs for crises and the growth of foreign lending is a robust one. This observation motivates a model which could generate significant output drop with a small change in the international capital market. However, a model as such is not satisfactory either. A large literature on "growth and finance" indicates that the development of financial market promotes growth. So the challenge is to reconcile these two literature in an unifying framework. The main theme of this paper is that financial development does improve growth by providing leverage, and yet it makes the local economy more sensitive and vulnerable to small changes in conditions in international capital market.

There are two apparent shortfalls in our analysis. The first one is the assumption that the user cost of capital in the domestic economy is perpetually below the world interest rate. This can not literally be true. Traditional analysis of international borrowing typically assumes decreasing returns so that the marginal product in poor countries is above the world average, resulting in temporary capital inflow which would be reversed in the process of development. We could enrich our model by assuming that there is an initial region of decreasing returns, as in the model of Jones and Manuelli (1990). The world interest rate is then assumed equal to the long run marginal product of capital of the economy. In this model, the country only borrows in its transition to the constant growth equilibrium. The alternative model, while certainty more realistic, is more clumsy to analyze but has few additional insights for our purpose with the possible exception that it may deliver better quantitative predictions.

The second defect in our analysis is that we have not explicitly modelled uncertainty in the world interest rate (perhaps credit ratings as well). Again doing so would certainty have resulted in a more realistic model. Upon a moment of reflection, it seems that the analysis would not have changed in any important ways though. When the debt is of duration $dt \simeq 0$, the country only borrows when $r < A - \delta$. It can not be worse off and the average growth rate must be higher with international borrowing than without. And when r rises above $A - \delta$, it simply stops borrowing. It is only when the country has to commit to long term debt with a fluctuating interest rate that there would be substantial change in the analysis.

References

[1] Bencivenga, Valerie R.and Bruce Smith, "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*; 58 (1991):195-209.

[2] Boyd John H. and Bruce Smith, "Capital Market Imperfections, International credit markets, and nonconvergence," *Journal of Economic Theory*, 73 (1997):335-64.

[3] Calvo, Guillermo and Enrique Mendoza, "Mexico's Balance-of-Payments Crisis: a Chronicle of a Death Foretold," *Journal of International Economics*, 41 (1996):235-264.

[4] Cole, Harold and Timothy Kehoe, "A Self-Fulfilling Model of Mexico's 1994-1995 Debt Crisis," *Journal of International Economics*, 41 (1996):309-330.

[5] Frankel, Jeffrey and Andrew Rose, Currency Crashes in Emerging Markets: an Empirical Treatment," *Journal of International Economics*, 41 (1996):351-366.

[6] Friedman, Milton and Anna Schwartz, *A Monetary History of the United States, 1867-1960*, Princeton: Princeton University Press, 1963.

[7] Greenwood, Jeremy and Boyan Jovanovic, "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, 98 (1990):1076-1107.

[8] Jones Larry E. and Rodolfo E. Manuelli, "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, 98 (1990):1008-1038.

[9] Keynes, John Maynard, *The General Theory of Employment, Interest and Money*, London: Macmillan, 1936.

[10] King, Robert G. and Ross Levine, "Finance and Growth: Schumpeter Might Be Right," *Quarterly Journal of Economics*, 108 (1992):717-37.

[11] Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy*, 105 (1997):211-248.

[12] Kiyotaki, Nobuhiro, "Credit and Business Cycles," *Japanese Economic Review*, 49 (1998):18-35.

[13] Levine, Ross, "Financial Development and Economic Growth: Views and Agenda," *Journal of Economic Literature*, 35 (1997):688-726.

[14] Levine, Ross and Sara Zervos, "Stock Markets, Banks, and Economic Growth," *American Economic Review*, 88 (1998):537-558.

[15] Long, Ngo Van and Kar-yiu Wong, "Endogenous Growth and International Trade: a Survey," in Bjarne Jensen and Kar-yiu Wong (ed.), *Dynamics, Economic Growth, and International Trade*, Ann Arbor: University of Michigan Press, 1997.

[16] Lucas, Robert, "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4 (1972):103-124.

[17] McKinnon, R.I., *Money and Capital in Economic Development*, Washington, D.C.: Brookings Institution, 1973.

[18] Obstfeld, Maurice and Kenneth Rogoff, *Foundations of International Macroeconomics*, Cambridge: MIT press, 1996.

[19] Otker, Inci and Ceyla Pazarbasioglu, "Speculative Attacks and Macroeconomic Fundamentals: Evidence from Some European Currencies," *European Economic Review*, 41 (1997):847-860.

[20] Rebelo, Sergio, "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99 (1991):500-521.

[21] Turnovsky, Stephen, "Endogenous Growth in a Dependent Economy with Traded and Nontraded Capital," *Review of International Economics*, 4 (1996):300-321.

[22] Turnovsky, Stephen, *International Macroeconomic Dynamics*, Cambridge: MIT Press, 1997.

[23] Turnovsky, Stephen and Pradip Chattopadhyay, "Volatility and Growth in Developing Economies: Some Numerical Results and Empirical Evidence," manuscript, University of Washington, Seattle, 1998.

[24] Turnovsky, Stephen and Partha Sen, "Investment in a Two-Sector Dependent Economy," *Journal of the Japanese and International Economies*, 9 (1995):29-55.

[25] Velasco, Andres and Roberto Chang, "The Asian Liquidity Crisis," New York University, manuscript, 1998.

[26] Wong, Kar-Yiu (ed.), The Asian Crisis: what has happened and why?, manuscript, University of Washington, Seattle, 1998.

Notes

[1]For instance, see McKinnon (1973).

[2]See Levine (1997), Levine and Zervos (1998) for more evidence and a survey of the literature.

[3]See Turnovsky (1997). For a dissenting view that foreign capital inflow may inhibit growth, see Boyd and Smith (1997).

[4]For instance, see Frankel and Rose (1996), Otker and Pazarbasioglu (1997), Wong (1998).

[5]See also Kiyotaki (1998).

[6]See Long and Wong (1997) for a discussion.

[7]In constrast, Turnovsky and Chattopadhyay (1998) do not impose an explicit bound on international borrowing but instead assume an upward sloping supply of funds schedule.

[8]We refer the readers to the discussion in GJ where this assumption is first introduced in models of finance and development.

[9]KM give an excellant discussion for the justification that creditors would not be willing to lend more than the value of the borrowers' tradeable assets.

[10]It may nevertheless do so through borrowing.

[11]We can rewrite the borrowing constraint in general form as $b(\tau) \leq \mu k(\tau)$ where $\mu < 1$. The amount of debt must only be a fraction of capital stock for there to be a bound on borrowing. To see this, note that if $\mu = 1$, (6) may be rewritten as

$$x(\tau) = (A - \delta - r) k(\tau) - c(\tau) + x(\tau).$$

This means that so long as the household sets $c(\tau) = (A - \delta - r) k(\tau)$, any value of $x(\tau)$ will satisfy the budget constraint and the household may borrow an infinite amount. The borrowing constraint is of no effect in this case because borrowing and the capital stock to be used as collateral can be increased one-for-one if $b = k$.

[12]That is why we *did not* form the Hamiltonian at T and evaluate the associated first order condition above.

[13]See Obstfeld and Rogoff (1996, chapter 2).

[14]The subscript a in the value function stands for autarky.

[15]Note that the first term in the right side of (25) is just the ratio of $m_r A - 1$ and $A - \delta$ which can be interpreted as the respective productivities with borrowing and without borrowing.

[16]For instance, see Calvo and Mendoza (1996), Cole and Kehoe (1996), Velasco and Chang (1988) and the reference therein.

[17]Strictly speaking, the model implies that all domestic production should shut down in this case, with all resources lent to the outside world. For the sake of argument, we assume that the household's equity can not be sold in short notice to prevent this extreme prediction of the model.

[18]Details available in an appendix available upon request.

[19]When $r' > A-\delta$, all international borrowing would halt, $\gamma_c = \gamma_c^a$. This imposes an upper bound on how much growth can decline.

[20]With the exception when $r' > A - \delta$ in which case the growth rate resorts to its autarkic level $\gamma_c^a = A - \delta - \rho$.