DUAL EXCHANGE RATE REGIME WITH FRAUDULENT LEAKAGE AND ITS UNIFICATION: THE CASE OF CHINA

Jian Xu

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(Preliminary. Comments Most Welcome.)

1. Introduction

Dual exchange rate regime is fairly popular after the Bretton-Woods system was failed in later 1960s. Many of the western countries used it as a transition to move from a dollar based fixed exchange rate regime to a market determined floating exchange rate regime. After over twenty years of adjustment and readjustment, almost all the major economies had accepted managed exchange rate system. However, a large number of developing countries still adopt separated exchange rates with a relatively fixed official exchange rate for government interfered economic transactions, often for commodity transaction, and relatively flexible exchange rate for self-balanced market transactions such as financial market transactions. In many more countries, although there is no legal flexible exchange market, but the illegal underground black foreign currency exchange market did exist. In such situation, two foreign exchange markets simultaneously remain in a country and it is often named as dual exchange rate market. In the so called dual exchange rate regimes, central authority usually intervenes in the commercial transactions to maintain a stable current account in foreign trade while leaving the financial transactions to a floating exchange rate regime to ensure capital account equilibrium by the market force.

The World Currency Yearbook reported that in 1989 total 113 countries under various systems of foreign exchange controls maintained multiple currency markets during middle of the 80's. In 1998 there are still 90 countries with foreign exchange rate control. Among them, twenty-four countries declared to adopt dual or multiple exchange rates. As commonly known, in the rest sixty- six countries, which adopt fixed exchange rate regime, non-official exchange market often exists in various illegal formats. (See exchange arrangements and exchange restrictions, IMF, 1998)

Along with the foreign trade reform in China, the policy adjustment by the state in exchange rate regime displayed tremendous similarities as the western countries and developing countries did. For over three decades from 1950s to the eve of 1980s, China adopted a fixed exchange rate with Yuan/$ rate equaled RMB1.5 to the dollar. An internal settlement rate of RMB 2.8 to the dollar was introduced in 1981 by the government in the attempt to remedy the substantial overvaluation. From that time on the authorities began a steady and gradual process to devaluate the official exchange rate. In 1984 the official rate reached its internal rate of 2.8. The official rate further increased to 3.2 in mid-1986, and then devalued the currency by 15 percent in the next year, taking the rate of RMB3.68 per dollar. Considering the growing pressure from illegal black market for pressing the RMB further devaluating, beginning in late 1986, the state introduced a formal secondary market for foreign exchange, often named as “the swap market”, where the exchange rate was permitted to be much higher than the official fixed rate and to be floating in reacting to the market demand and supply. The co-existence of official exchange and swap exchange market is a form of dual exchange rate market. In December 1989, the official exchange rate was devalued by 21.2 percent while the swap rate reached up to over RMB 6/dollar. When the time moved into the 1990s, there were several more miner devaluations to bid up the official exchange rate up to RMB 5.2 per dollar following the preceding adjustment in the swap market. Then in January 1994 the official exchange rate and the swap rate were unified at the near secondary market rate of RMB 8.7 per dollar. Some moderate appreciation holds the nominal exchange rate fluctuated within a narrow rage around 8.2 since 1995. The above description shows, from 1980 to 1994 China had experienced a transition from fixed exchange rate to a dual exchange rate system, which ended up with a unified managed-floating exchange rate (with the variety of exchange rate in Yuan/$ of only 3.6% in eight years from 1994 to 2002, this would rather is considered as a managed fixed exchange rate) in 1994. Currently, Chinese firms have ready access to foreign exchange at the official rate to pay for imports. Foreign owned and joint venture firms can convert domestic currency profits into foreign exchange and remit them abroad. It has evidence that the commodity market is dominated by official exchange rate and is well integrated with the outside market. But the opportunity for Chinese firm and individual to hold foreign-currency denominated financial assets, such as stock and bonds is eliminated. Foreign firms and individuals are limited to buying foreign-currency denominated shares of Chinese companies known as B share. In the meantime foreign firms and individuals are precluded from purchasing Chinese-currency denominated financial assets, such as government bonds or stocks referred as a share. It is apparently that the government of China only permits the financial market a shallow integration into the world market to prevent the financial risks. However, transactions conducted in financial market are based on floating exchange rate determined by market demand and supply.

One of the critical steps the state of China took in foreign trade reform during the middle of 1980s was to allow exporters to increasingly retain large share of their foreign exchange earnings. Exporters and importers were also allowed to access swap market to sell or purchase foreign exchange on the swap rate. Thus, importers may gain extra profit by overvalue their import at the official exchange rate and sell the extra exchange into swap market at higher swap rate. Exporters could also seek for extra profit by undervalue their exports, and sell the extra foreign exchange earnings in swap market. The incentive of fraudulent arbitrage caused overwhelmed illegal leakage between the two markets.

Driven by my interest in the recent two decades history of the movements in China's foreign exchange market, I have attempted to incorporate dual exchange rate and fraudulent leakage into an optimization framework, so as to study the dual market phenomena rooted with individual rational economic behavior. Eventually, as we have seen, all dual track exchange market has to transfer to a unified exchange market. The process of transition is full of risk and some countries even falling into turbulence. I intend to frame the dual track system into a theoretical model, which can be used to study the regime transition for choosing the best possible transitional path.

Although the specific operation of dual exchange rate regime might vary across countries, the basic structure is similar. It involves an official (commercial) rate market in which the exchange rate is determined by authority and is relatively appreciated, and a secondary (financial or free) exchange market where the exchange rate is determined by market forces, and is relatively depreciated. An important feature of the dual exchange rate market is that the illegal transactions from official market to the free market in response to exchange rate differentials are very likely to take place. There are indications to show that arbitrage activities between the two markets is not just an interesting phenomenon, it has a practically significance as well.

The literature on dual exchange rate systems may fall into two groups. In one, Authors adopt the assumption that the markets in a dual exchange system can be completely separated from each other. In other words, there are no inter-market transactions. [See Argy and Porter (1972), Marion(1981), Flood and Maion(1982), Flood and Marion (1983), Aizenman(1985), Guidotti and Vegh (1988)]. With this assumption a dual exchange rate regimes would be effectively insulated from outside disturbances. This would allow the system to pursue a monetary policy independently from the external disturbances. While the flexible exchange rate maintains a balance in capital account, the current account is buffered by the international reserves. This could be true only if complete segmentation exists.

The second group, mostly new literature, recognizes the illegal leakage between the fixed exchange and floating exchange markets. In a dual exchange rate system the financial and commercial market are very difficult to keep absolutely separated because the divergence between the two exchange rates prevailing in the two markets creates great incentives to arbitrage from inter-market transactions. Economic agent would seek profit by misrepresenting capital account transactions as payments associated with trade or vice versa (Flood, Perraudin and Vitale, 1998). Early literature related with inter-market leakage can be found in the works of Bhandari and Decaluwe (1987), Lizondo (1987), Daniel (1988). Bhandari and Vegh (1990), which studied how systematic over or under-invoice current account transactions would disguise capital account transactions as trade-related transactions and hence perform arbitrage activities.

Agenor and Debecgue (1991) and Agenor (1992) show how reserve losses from leakage in a dual exchange rate system may lead to the collapse of a fixed exchange rate. As early as the later 1970s, Rodriguez (1978) first noticed the instability feature in dual exchange rate system and later Dornbusch(1983) and Kamin(1993) addressed the issue of steady-state cycles in an informal way. Flood, William and Paolo(1998) revisited this topic using a formal conventional linear dynamic model. They analyzed the reserve cycles associated with the leakage. However, none of the research has ever touched the key point in dual exchange rate system, that is the insatiability nature of the system and how an unstable system could ever maintain in a relatively stable status, even with a cyclical feature.

The presence of cross-market transactions implies that despite the separation rules imposed by the authorities, the capital account is not in zero balance, and net accumulation or decumulation of foreign assets is still possible. Moreover, the effectiveness of the dual rate system to maintain the insulation of domestic market is actually partially offset by the leakage.

The dual exchange rate system, especially corporating with illegal transactions, has not been explored much under the representative agent optimization framework. To my knowledge, analysis covering the dual exchange rate regime incorporating optimization behavior (see Obstfeld (1986), Gudotti (1988)) all adopted the assumption of a perfect segmentation between the official market and the secondary market.

Instead of assuming an official exchange rate to be fixed by the authority, as was assumed by most authors, we apply a crawling peg exchange rate regime to the official market and keep the secondary market as a floating exchange rate regime. These improvement make the model closer to the real economy in which official rate is not simply fixed without reaction to any external disturbances. The model constructed here is based on the work of Guidotti and Vegh (1988) and benefits from Calvo's (1987) approach to setting the individual's wealth constraints. A fourth order differential equations system is derived from the optimization framework. This system would characterize the dynamic movement among individual’s wealth distribution and government macro policy change. It allows the examination of the effects of some exogenous disturbances to the system. Anticipated and unanticipated responses are analyzed. The results we obtained here extend Flood, Perraudin and Vitale's study in cyclical phenomena in dual exchange rate system. We show that the fraudulent leakage not only generates cycles in reserves, it also performs critical function in maintaining an unstable system in a relatively long time period. As a common trend, most countries, which adopted dual system in early 80's, had transformed, in some extents, to unified system by later 80's. The model developed here also allows us to examine the transition from dual system to (1) a unified floating exchange rate system, and (2), a unified crawling peg system. A complete dynamic solution from the model is obtained, which allows us to examine some vague issues in transitional process.

The rest of the paper is organized as follows: Section one has developed an representative agent optimization model which links fraudulent arbitrage flows to the exchange rate differential in a dynamic setting. In section two we analyzed the effects of exogenous shocks including a government spending and an inflationary disturbance. Section three discusses the unification of the dual system. We compared two different possible options and their results. Section four concludes the discussion with several possible extensions.

2. The Dual Exchange Rate Economy with Leakage

In this section we consider a small open economy with a dual exchange rate system. In this economy commercial transactions are settled in the official exchange rate market at the official exchange rate e, which is depreciated at a constant rate p (a case of crawling peg exchange rate system) controlled by the central bank. The capital transactions are settled in a non-official exchange rate market at a floating exchange rate s, which is determined by the market force without government intervention in this market. Both e and s are defined in unites of domestic currency per unit of foreign currency.

As indicated by Deniel Gros (1990):" the existence of a dual exchange rate offers arbitrage opportunities to economic agents in the sense that they would like to buy foreign exchange at the lower rate and sell it at the higher rate." The following model aimed at describing those features.

We assume that this economy enjoys a constant endowment of an exportable good y. However the domestic consumer does not consume the endowment, the consumption good c is imported from foreign market. For simplicity, we assume that the relative price of exportable good in terms of importable good is set equal to unity.

As being usually assumed, there is only one goods in the world, its domestic price P and the world price P^* is linked by the purchasing power parity, $P = eP^*$. It follows that the world price P^* is set equal to unity for simplicity. So $P = e$, and $\dot{P} = \dot{e}$, meaning that $\dot{e}$ is denoted as inflation rate. In an economy with credit system not being highly developed, cash is the most commonly used intermediary way for consumption. Especially for a developing country like China where financial system has not been well developed, consumers mostly hold money in terms of cash for consumption. It is natural to introduce the cash-in-advance constraint, namely, $M = aPc$, into the model here. Here M is the nominal money balance and C is the quantity of real consumption. It can easily be converted to a real term, $m = ac$, where m is the real money and $a > 1$. Let the world interest rate be fixed at r, and the financial rate s be completely flexible, therefore the real quantity of external assets-specifically here, the foreign bonds held by the private sector is fixed at a constant level.

Arbitrage Activity

This dual exchange rate system is characterized by the possibility of cross-transaction or leakage between the two exchange rate markets. The basic feature for fraudulent behavior as indicated above is that the economic agents attempt to buy foreign exchange at the lower rate in trade market and sell it at the higher rate in financial market. One of the important way to circumvent the rules designed to keep the two markets separate would be a fraudulent activity of overinvoice and underinvoice in both exports and imports. The illegal income from fraudulent activities could then be repatriated at the market with a higher rate.

An interesting point we addressing here is that the fraudulent leakage may cause domestic people to buy and sell foreign bonds. Although the official rate e is depreciated at the rate of p (actually we consider the case of crawling peg), e could not reflect the change in market as quickly as the financial rate S. This creates an opportunity for people to gain extra profit by overinvoice or underinvoice in foreign trade. If the official rate e is relatively depreciated compared to the market rate s, the individual will take overinvoice in export and under invoice in import. If in the case that the e is relatively appreciated vis-à-vis S, then individual will underinvoic in his export and over invoice in import. In this model we define L_t as the measurement of the real amount of leakage at time t. Henceforth the gain from the leakage would be a product of the difference between official rate e and the market rate s and the total amount of leakage L_t. We define that the total amount of leakage is the sum of the leakage in both export and import. When the market rate s is greater than the official rate e, the leakage L_t is positive; L_t is negative if s is smaller than e.

We indicate that under our assumptions, the change in the stock of foreign bonds held by individual is only caused by the fraudulent leakage between the dual track exchange market with the exchange earnings from export and import. Also we need to be aware that the illegal activity of making leakage is not free of cost. Government prevents leakage by impose severe punishment to fraudulent behavior. For simplicity we assume that the cost is positively correlated to the amount of leakage L. Specifically, we define the cost function is quadratic, that is

$$g(L) = \frac{e\,L^2}{2K}$$

It implies that the benefit gained from faudulent leakage has an effect of diminishing marginal return associated with the quantity of leakage.

Now we can define the profit function derived from fraudulent under-or-over invoice as:

$$p(L_t) = (s_t - e_t)L_t - \frac{e L_t^2}{2K} \tag{2.1}$$

where *s, e* and *L* are defined before. The second part of (2.1) is the cost of illegal transactions. *K* is set such that $0 < K < 1$, to keep the cost at certain level. *K* can be interpreted as an indirect transaction cost.

Consumer Utility Optimization

The representative consumer owns his wealth $A = M + sb$ in nominal term, or $a = m + qb$ in a real term, where *M* is nominal money, *b* is the foreign real bonds and $q = s/e$ is the real price of bonds while $m=M/p$ is the real term of money. The representative consumer maximize his utility under the constraint of his wealth accumulation:

$$\dot{A}_t = e_t y_t + P(L_t) + re_t b_t - e_t c_t + e_t t - M_t p + \dot{s}_t b_t \tag{2.2}$$

where $P(L_t)$ is defined in (2.1), y_t is the endowment, b_t is the foreign bonds, M_t , the money balance, τ denotes the real transfers from the government to the individual. $\dot{s}_t b_t$ is the term of capital gain, $re_t b_t$ denotes the interest proceeds on holding the foreign assets and $M_t p$ is the inflation tax. Let $a = A_t/e$ be real term of wealth and notice $\dot{a} = \dot{A}/e - ap$, we can rearrange equation (2.2) in a real term as

$$\dot{a} = y_t + \frac{P(L_t)}{e} + (r + \dot{q}_t)b_t - c_t + t - m_t p - a_t p + pq_t b_t \tag{2.3}$$

Recalling the cash-in-advance constraint and the definition of wealth,

$$A_t = M_t + s_t b_t$$

and it's real term

$$a_t = m_t + q_t b_t$$

(2.3) can be rewritten as

$$\dot{a}_t = y_t + \frac{P(L_t)}{e_t} + r_t b_t - c_t + t_t - (2ap)c_t \tag{2.4}$$

further more, substituting with CIA constraint in Chapter (1) and the definition of wealth into (2.4), we obtain:

$$\dot{a}_t = y_t + (L_t(q_t - 1) - \frac{L_t^2}{2K}) + r a_t - ((r_t + 2p)a + 1)c_t + t \tag{2.5}$$

where r_t is defined from $\dot{q}_t = r_t q_t - r$. It is clear that from equation (2.4) r_t can be interpreted as the real interest return on bonds. However, the second form of wealth constrain equation (2.5) is more useful in solving the optimization problem.

Now the optimization problem that the representative consumer faces is to choose $c_{t,}\ b_t, m_t$ *and* L_t to maximize his utility over time from 0 to ∞, we state this problem as:

$$\underset{c_t, m_t, b_t, L}{Max} \int_0^\infty u(c_t) e^{-dt} dt$$

Subject to equation (2.5). where d stands for the time discount rate. We drop the subscript t from this point on the purpose of notational simplicity. The optimal conditions of the problem are listed as follows:

$$U_c = l((r + 2p)a + 1) \tag{2.6}$$

$$\dot{l} = l(d - r) \tag{2.7}$$

$$\dot{q} = rq - r \tag{2.8}$$

$$L_t = K(q - 1) \tag{2.9}$$

Equation (2.6) is one of the familiar F.O.C, which equates the marginal utility of consumption with the product of marginal utility of wealth l and the effective price of consumption good which is the sum of market price (unity) and the opportunity cost of holding a units of money. Equation (2.7) describes the evolution of the shadow price l. Equation (2.8) indicates the dynamic path of the real price of bond q. Equation (2.9) gives the optimal level of leakage resulting from the arbitrage activity in export and import transactions.

Using the definition of wealth and the government budget constraint and money supply equation,

$$t = rf + mp - g \tag{2.10}$$

$$m = f + D/p = f + D/e \tag{2.11}$$

where f denotes the foreign reserve, g is the government spending and D stands for the domestic credit. Both g and D are exogenously determined. p is the domestic price. Since we assume the foreign price equal to unity, thus the p is equal to e by the P.P.P. assumption. Combining the above two equations (2.10) and (2.11), we obtain

$$t = r(m - \frac{D}{p}) - g + mp \tag{2.12}$$

This is useful for changing individual wealth constraint (2.2) into a balance of payment equation in terms of *M*. With (2.12), (2.2) can be described as:

$$\dot{M} = ey + L(s-e) - \frac{eL^2}{2K} + reb - ec + r(M - D) - eg \tag{2.13}$$

substitute (2.9) into the above equation, we obtain

$$\dot{M} = ey + \frac{K}{2}e(q-1)^2 + re\,b - (\frac{1}{a} - r)M - rD - e\,g$$

Its real term is:

$$\dot{m} = y + \frac{K}{2}(q-1)^2 + rb + (r - \frac{1}{a} - p)m - rd - g \tag{2.14}$$

where d=D/*e* is the real value of domestic credit.

From equation (2.14), we can see the change of money stock is now affected by q, therefore, equivalently by financial rate s. Thus the disturbances from external market could influence the domestic monetary policy through the financial exchange rate. Affected by these external shocks, domestic monetary policy could no longer be independent.

Combining (2.6) and (2.7) we derive the second differential equation, which describe the evolution of the real domestic interest rate:

$$\dot{r} = \frac{U_{cc}}{a^2 l}(\dot{m}) - (d - r)(r + 2p + \frac{1}{a}) \tag{2.15}$$

Considering the equation (2.7), (2.15) has another useful version of description:

$$\dot{r} = \frac{U_{cc}}{a^2 l}(\dot{m}) - \frac{1}{l\,a} U_c (d - r) \tag{2.15'}$$

This illustrates that the change of domestic real interest rate is related to marginal utility associated with consumption c and the first and second order derivatives of utility with respect to consumption. This reflects that individual preference matters in a macro problem.

Under the assumption that the foreign price level and interest rate of the foreign bonds are fixed, the aggregate level of the stock of foreign bonds is fixed. In equilibrium the private holding of bonds can only change as a result of illegal transaction; therefore, the change of bonds equal to the amount of leakage:

$$\dot{b} = K(q - 1) \tag{2.16}$$

Rearranging the whole system we obtain the following:

$$\begin{cases} \dot{m} = y + \dfrac{K}{2}(q-1)^2 + rb + (r - \dfrac{1}{a} - p)m - rd - g \\ \dot{r} = \dfrac{U_{cc}}{a^2 l}(\dot{m}) - (d - r)(r + 2p + \dfrac{1}{a}) = \dfrac{U_{cc}}{a^2 l}\dot{m} - \dfrac{(d - r)}{l\,a} U_c \\ \dot{q} = rq - r \\ \dot{b} = K(q-1) \\ \dot{l} = l(d - r) \end{cases} \tag{2.17}$$

There are five differential equations of m, r, q b and l and are corresponding to exogenous variables g, d, $\mathbf{d}$, p and r. Setting $\dot{r}, \dot{q}, \dot{b}$ *and* $\dot{m}$ equal to zero, enables us to solve for the steady state $\bar{l}, \bar{m}, \bar{r}, \bar{q}$ *and* $\bar{b}$.

Let $\dot{l} = 0$, we know $d = r$ and $l = \bar{l}$.

Setting $\dot{b}=0$, we obtain $\bar{q} = 1$. It shows in steady state, official exchange rate must equal to the market rate. This is the necessary condition of stopping fraudulent arbitrage in equilibrium, and is also an appealing feature since the model confirms with the intuitive motion that any divergence between e and S is not sustainable in the long run equilibrium. Logically, once the free exchange rate S is equal to the official exchange rate e, then the foreign bonds reserve will remain at certain level $\bar{b}$ because lacking the incentive of leakage. $\dot{q} = 0$ implies that $r = r = d$. Noticing that m=M/e and setting $\dot{r} = 0$, we can see the condition for steady state is $\dot{m} = 0$. Equivalently, it is $\dot{M} = pM$. It tells that the growth rate of nominal money is also p which is equal to inflation rate. In other words, while there is a stable growth rate of nominal money, which is as same as the crawling rate p, the growth rate of real money m must equal to zero. This should not be surprising. For a crawling peg case, keeping the money growth rate exactly the same as the rate of inflation is intuitively an acceptable condition to keep the economy at a stable level. For memorandum, we rewrite the steady state solutions here:

$$\begin{cases} \bar{q} = 1 \\ \bar{r} = r \\ \dot{\bar{b}} = 0 \\ (p + \frac{1}{a} - r)\bar{m} = y + r\bar{b} - rd - g \end{cases} \tag{2.18}$$

given initial conditions, we can solve both $\bar{m}$ *and* $\bar{b}$. Meanwhile we can obtain the steady state Lucas $\dot{m} = 0, \dot{r} = 0, \dot{q} = 0, \dot{b} = 0$ and the slopes of these curves:

$$\bar{m} = \frac{1}{(p + \frac{1}{a} - r)}(y + r\bar{b} - rd - g) \tag{2.19.1}$$

$$\bar{r} = -\frac{U_{cc}}{aU_c}\bar{m}(r - \frac{1}{a} - p) + \{r - \frac{U_{cc}}{aU_c}(y + r\bar{b} - rd - g)\} \tag{2.19.2}$$

$$\bar{r} = \frac{r}{\bar{q}} \tag{2.19.3}$$

$$\bar{q} = 1 \tag{2.19.4}$$

These four steady state solutions are important to describe the dynamic change caused by any disturbances of government policy and inflation change. One interesting result we obtained here is: In steady state, $\bar{q} = 1$. This implies that the necessary condition for steady state is

$\bar{s} = \bar{e}$. Apparently if $\bar{s} > or < \bar{e}$, there is no steady state. So the dual exchange rate system is an unstable one.

By linearlizing the differential equations in (2.2.17) around the steady states, we have the constant coefficient linear differential equation system:

$$\begin{pmatrix} \dot{m} \\ \dot{r} \\ \dot{q} \\ \dot{b} \end{pmatrix} = \begin{pmatrix} r - \frac{1}{a} - p & 0 & 0 & r \\ \Phi(r - \frac{1}{a} - p) & r + 2p + \frac{1}{a} & 0 & \Phi r \\ 0 & 1 & r & 0 \\ 0 & 0 & K & 0 \end{pmatrix} \begin{pmatrix} m - \overline{m} \\ r - \overline{r} \\ q - \overline{q} \\ b - \overline{b} \end{pmatrix} \tag{2.20}$$

where $\Phi = \frac{U_{cc}}{a^2 l} < 0$.

We have two obvious conclusions from the coefficient matrix:

- The trace of (A) = 3r + π > 0, so there is at least one positive eigenvalue.
- The determinant $|A|$ = 0, it implies that one of the eigenvalue is equal to zero.

This result of the determinant is true given the assumption that *1/a - r > 0*. The implication of it is *C -rm > 0*. It means that in this economy, consumption level exceeds the interest income from the money stocks; therefore, this is a fairly reasonable assumption.

Appendix (II.1) demonstrates the determination of signs of the eigenvalues of the determinant $|A|$.

Conclusion: For the determinant $|A|$ in (2.2.20), given that *1/a - r > 0*, and knowing the trace of it is greater than zero, and the determinant of it is equal to zero, we can prove that there is only one negative characteristic root $l_1 < 0$, one zero characteristic root and two positive characteristic roots.

The general solution of (2.19) is in the form of:

$$\begin{pmatrix} m \\ r \\ q \\ b \end{pmatrix} = \begin{pmatrix} \overline{m} \\ \overline{r} \\ \overline{q} \\ \overline{b} \end{pmatrix} + \begin{pmatrix} u_{11} & u_{21} & \cdots & u_{41} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ u_{14} & u_{24} & \cdots & u_{44} \end{pmatrix} \begin{pmatrix} K_1 e^{l_1 t} \\ K_2 e^{l_2 t} \\ K_3 e^{l_3 t} \\ K_4 \end{pmatrix} \tag{2.21}$$

where the vector $\vec{u}_i = (u_{i1} \quad u_{i2} \quad u_{i3} \quad u_{i4})$ is the eigenvector associated with eiganvalue l_i.The conclusion ensures that, given initial condition, there exists a unique convergent continuous dynamic path which satisfies the differential equation system (2.19) provided that the arbitrary scalar K_i = 0 (i = 2, 3,). The convergent solution of (2.20) can be written as

$$\begin{cases} m = \overline{m} + u_{11} K_1 e^{l_1 t} + u_{41} K_4 \\ r = \overline{r} + u_{12} K_1 e^{l_1 t} + u_{42} K_4 \\ q = \overline{q} + u_{13} K_1 e^{l_1 t} + u_{43} K_4 \\ b = \overline{b} + u_{14} K_1 e^{l_1 t} + u_{44} K_4 \end{cases} \tag{2.22}$$

The Eigenvalue-vector solved from the equation $(A - l_i I)\,\vec{u}_i = 0$ is:

$$\begin{cases} (r - \frac{1}{a} - p - l_i) u_{i1} + r u_{i4} = 0 \\ \Phi (r - \frac{1}{a} - p) u_{i1} + (r + 2p + \frac{1}{a} - l_i) u_{i2} + \Phi r u_{i4} = 0 \\ u_{i2} + (r - l_i) u_{i3} = 0 \\ K u_{i3} - l_i u_{i4} = 0 \end{cases} \tag{2.23}$$

where i = 1,4. For $l_1 < 0$, it is:

$$\begin{cases} (r - \frac{1}{a} - p - l_1) u_{11} + r u_{14} = 0 \\ \Phi (r - \frac{1}{a} - p) u_{11} + (r + 2p + \frac{1}{a} - l_1) u_{12} + \Phi r u_{14} = 0 \\ u_{12} + (r - l_1) u_{13} = 0 \\ K u_{13} - l_1 u_{14} = 0 \end{cases} \tag{2.24}$$

while for $l_4 = 0$, it is:

$$\begin{cases} (r-\frac{1}{a}-p)u_{41}+ru_{44}=0 \\ \Phi(r-\frac{1}{a}-p)u_{41}+(r+2p+\frac{1}{a})u_{42}+\Phi r u_{44}=0 \\ u_{42}+ru_{43}=0 \\ Ku_{43}=0 \end{cases} \quad (2.25)$$

From (2.24), we also can solve the following relationships:

$$\begin{aligned} u_{12} &= \frac{(r-l_1)l_1(r-1/a-p-l_1)}{K}u_{11} \\ u_{13} &= -\frac{l_1(r-1/a-p-l_1)}{Kr}u_{11} \\ u_{14} &= -\frac{(r-1/a-p-l_1)}{r}u_{11} \end{aligned} \quad (2.26)$$

From (2.25) it is easy to see $u_{42}, u_{43}=0$, and

$$u_{44}=\frac{p+1/a-r}{r}u_{41}.$$

According to the theory in linear algebra, for a homogeneous equation system, if it has solution, it has more than one solution depending upon the arbitrary value(s) given to at least one variable. For equations (2.21) and (2.22), the general solution is

$$\begin{cases} m=\overline{m}+u_{11}K_1e^{l_1t}+u_{41}K_4 \\ r=\overline{r}+\frac{(r-l_1)l_1(r-1/a-p-l_1)}{K}u_{11}K_1e^{l_1t} \\ q=\overline{q}-\frac{l_1(r-1/a-p-l_1)}{Kr}u_{11}K_1e^{l_1t} \\ b=\overline{b}-\frac{(r-1/a-p-l_1)}{r}u_{11}K_1e^{l_1t}+\frac{p+1/a-r}{r}u_{41}K_4 \end{cases} \quad (2.27)$$

where $u_{11,}K_1$ *and* u_{41}, K_4 are all arbitrary scalars. For the purpose of convergence, we let K_4 equal to zero. Also the solutions need to converge to a definite point. Since here $u_{i1}K_i$ is undeterminable by simply using the initial condition, we have to set $u_{41}K_4$ equal to zero. The solution of (2.27) is simplified as:

$$\begin{cases} m = \overline{m} + u_{11}K_1e^{l_1t} \\ r = \overline{r} + \dfrac{(r-l_1)l_1(r-1/a-p-l_1)}{K}u_{11}K_1e^{l_1t} \\ q = \overline{q} - \dfrac{l_1(r-1/a-p-l_1)}{K}u_{11}K_1e^{l_1t} \\ b = \overline{b} - \dfrac{l_1(r-1/a-p-l_1)}{K}u_{11}K_1e^{l_1t} \end{cases} \tag{2.28}$$

From (2.28) we can derive:

$$\begin{cases} \dfrac{m-\overline{m}}{r-\overline{r}} = \dfrac{1}{q_1} < 0, \text{ where } q_1 = \dfrac{(r-l_1)l_1(r-1/a-p-l_1)}{K} < 0; \\ \dfrac{m-\overline{m}}{q-\overline{q}} = \dfrac{1}{-q_2} > 0, \text{ where } q_2 = \dfrac{l_1(r-1/a-p-l_1)}{Kr} < 0, \\ \dfrac{m-\overline{m}}{b-\overline{b}} = \dfrac{1}{q_3} < 0. \text{ where } q_3 = -\dfrac{r-1/a-p-l_1}{r} < 0. \end{cases} \tag{2.29}$$

The solution (2.28) and (2.29) are derived from equation (2.21) and (2.22), the linealized differential equation system around their steady states. Thus (2.28) representing a solution of the system (2.20) and (2.29) shows the stable arms of the saddle path. The signs of the above slopes are determined, which describes the tendency of movement of the variables in their dynamic adjustment process. The saddle path in m and r space is a downward sloping line. The saddle path in m and q space is an upward sloping curve. Finally, the saddle path in m and b space is also a downward sloping curve. Knowing those relationships, we can now start the dynamic analysis of the above variables under disturbances from macro-policy change or inflation change.

3. Dynamic Analysis of the exogenous disturbances.

In this section we will analyze the effects of exogenous disturbances of change in government spending g, the domestic credit D and official exchange rate e on m, ρ, q and b.

Increasing in Government Spending

(A) Unanticipated increase of g.

Let's consider an unanticipated and permanent rise in government spending g. For an increase in government spending from $\overline{g} \rightarrow \overline{g}'$, from (2.18)

$$\frac{\partial \overline{m}}{\partial g} = \frac{-1}{p + \frac{1}{a} - r} < 0 \tag{3.1}$$

The real money balance decreases across the adjustment interval when the government spending expanding.

It is easy to show that the steady state of consumption will reduce in proportion to the change in real money balance due to the cash-in-advance constraint. The fact is that once the government raises its expenditure, the economy runs a current account deficit, which can be seen from the balance of payment equation (2.14). On the impact, the change in current account deficit is given by:

$$d\dot{m} = -\left(\frac{1}{a} + p - r \right) dm - dg \tag{3.2}$$

The effect of impact of an expansion fiscal policy manifests itself in a combination of an increase in price level and current account deficit. The deficit will be financed by lowering the consumption level over time until the system approaches to a new steady state. Since the cash-in-advance constrains relates the real money stock with consumption, a decreasing in m will bring the consumption down to a lower steady state level as well.

By equation (2.19), it is seen that the steady state level of the real price of bond q, and the real interest rate r, are fixed at constants 1 and r respectively acrossing the steady states. It means the steady state level of q and r will return to their original level at a new steady state after a transitional adjustment. Even though we cannot solve the steady state of bond reserve $\overline{b}$ from equation system (2.17), equation (2.16) demonstrates that the change of b is only determined by the change of q. Thus when q returns to its steady state 1, b will return to the original level $\bar{b}$ too.

In appendix (II.3), we proved the price level jumps up instantaneously right after the unanticipated rise in government spending. Intuitively, since in steady state, both the nominal money M and the inflation grow at the rate of p, a depreciation in domestic money will reduce the real money stock. This can be shown in figure (1)

After the initial jump, the price level will increase at the rate π once the system reaches a new steady state if the crawling peg rate remains the same.

Now we need to know the initial jump in q and r to derive the adjustment path, which the dynamic system will follow. According to the equation (2.28) we obtain the following:

$$q-\bar{q}=-\frac{l_1(r-1/a-p-l_1)}{Kr}(m-\bar{m}) \tag{3.3}$$

After the impact, steady state of real money balance will jump down from $\bar{m}$ to $\bar{m}$ '. Giving $\bar{q}$ remains at 1 under any circumstances, equation (3.3) indicates that $q>1$ after the policy shock because the coefficient of $m_0-\bar{m}$ is positive. In other words the initial jump in q is positive. We can interpret that although a increase in g causes the inflation, the financial market's response to the shock in real price of bonds is even more dramatic. This result is depicted on Figure (2).

Next, let's take a look at the first two equations in (2.28), which show the relationship between m and *? along the steady arms.*

$$r-\bar{r}=\frac{l_1(r-l_1)(r-1/a-p-l_1)}{K}(m-\bar{m}) \tag{3.4}$$

Because the coefficient of the right term is negative, by the same token, we can prove the initial impact of expansionary fiscal policy on r is negative. Thus on impact, r will jump down.

If there is no leakage, the bonds reserve is a constant over time, then according to the stable saddle path expressed by equation (2.29), after the initial jump q will return to its steady state along the stable arm. Equation (2.29) shows that the saddle path in "m-q" subspace is an upward sloping line. Therefore, the dynamic movement will be the following: On the impact, the real price of bond will jump up from the old steady state point $(\overline{m},1)$ to $(\overline{m},1+\Delta q)$ and then move back along the stable path to a new steady state point m with the same level of *q=1* but at a lower level of m. This is illustrated in figure (2.2).

But now at the presence of leakage, as long as q deviates from the steady state 1, the private sector raises the incentive for fraudulent action, which will affect the dynamic path of *q*. By looking at the phase diagram in "*q-b*" sub-space, we find that after the impact, the co-movement of q and b can be described as follows: On the impact, q jumps upward. But the model shows that arbitrage will reduce the difference between *s* and *e*, therefore to reduce the magnitude of q. This forces q to drop down towards 1. Intuitively, since during the time *q > 1*, illegal leakage accumulates the bonds reserves above $\overline{b}$. In the new steady state, *b* has to return to the original $\overline{b}$ level. So there must be an opposite force to induce individual changing from under invoice in export and overinvoice in import to an opposite one. This implies the real price of bond should not directly return to unity along the saddle path, but will decrease to undershoot the steady state, then increase near the "q-b" stable arm to return to $q=1$. Meanwhile, b increases when q is greater than 1 and then decreases after q undershooting the steady state. This dynamic can be seen in figure (2.3)

The implication of the above dynamic is that the real bonds reserve b is not a jump variable.

By using equation (2.13), we can show:

$$\frac{d\dot{M}}{dg} < 0 \tag{3.5}$$

An increase in g will instantaneously bring the nominal money growth down. With this in mind, we can describe the dynamic of q, b and r in terms of m:

On the impact, the real money jumps downward and then decreases across time until approach the new steady state $\overline{m}'$. *q* jumps up on impact and then slowly decreases to approach unity. It keeps moving downward to undershoot the steady state until reaches the stationary point, then start to climbs up back to the original steady state: $q=1$. This diagram is depicted on Figure (2.4). The bonds reserve b, according to figure (2.5), increases initially as $\overline{m}$ decreases to $\overline{m}$' in an adjustment movement. After reaches the stationary point, b decreases until it reaches the steady state $\overline{b}$. We describe these dynamic movements in figure (2.5) and (2,6).

Meanwhile, the real interest rate r decreases on impact and rises along the saddle path monotonically toward its steady state $\overline{r}$ = r. We depict this in figure (2.7).

Following the fact that the increase in government spending causes a deficit in current account because the government takes over more resources, fewer resources are available for consumer use. The deficit will be financed through time by lowering the steady state consumption level The representative consumer rationally predicts depreciation in official exchange rate, they also foresee decreasing in consumption. For preventing the inflation, they would like to buy foreign bonds. Logically, this will increase the real price of bonds and bid the financial rate s up even more than the reaction of official rate. The existence of leakage will generate the incentive for underinvoice in export and overinvoice in import, which in turn causes a further increase in the stock of foreign bonds due to the exporters and importers invest their illegal income of foreign currency in foreign financial assets. As far as q is greater than unity, it is profitable to sell these assets and repatriate at the financial rate. Along with the decrease of bonds reserves, the real price q drops down. When q reduces below unity, it will induce the incentive of overinvoice in exports and underinvoice in import, hence reverses the dynamic movement of b until it returns to steady state $\overline{b}$. Meanwhile, the domestic real interest rate r drops on impact as a response to the decrease on consumption. As q starts decreasing, r will climb up along the saddle path over time to induce the consumption path and finally returns back to the same equilibrium level of $\dot{x}(t)$, This gives the intuition of the movement in sub-spaces of "m-q", "m-b" and "m-ρ" depicted in figure (2.3), (2.5) and (2.6) respectively.

Besides the dynamic in the above three sub-spaces, it is also interesting to consider the "q

- ρ" sub-space. Using the second and third equations in (2.23) and (2.24), we obtain a downward sloping line:

$$q-\overline{q}=\frac{1}{r(r-l_1)}(r-\overline{r}) \tag{3.6}$$

Together with equations in (2.18.3) and (2.18.4), we can depict the phase diagram in figure (2.7), which shows the consistence with all the three diagrams from figure (2.4) to figure (2.6).

We conclude the above discussion by combining All the figures (2.4), (2.6) and (2.7) to establish a three-dimensional figure which depicts the dynamic change caused by a increase in government spending in a three dimensional space. This is the figure (2.8).

(B) Devaluation

Now we will consider the effects of a permanent and unanticipated "once for all" devaluation of domestic money, that is an increase in official rate e. According to the one price law, a rise in e implies an upward jump of the price level and thus reduces the real money balances instantaneously. However, from equation (2.14), we can see that the change of real money balance is positive over steady state,

$$\frac{d\overline{m}}{de}=\frac{rD}{e^2(p+\frac{1}{a}-r)}>0 \tag{3.7}$$

where D is the domestic credit in a norminal term. In the original steady state, on the impact, the price level jumps up instantaneously to induce an instantaneous drop in real consumption. After an adjustment interval, the real balance of money stock has to be raised proportionally. Knowing that $\overline{m}'>\overline{m}$ and the increase in domestic price, nominal money balance M will be higher in the new steady state. As a sluggish variable, the growth rate of M must exceed π across the adjustment period. Following the cash-in-advance constraint, consumption also falls when impact occurred, therefore the economy runs current account surplus during the adjustment period, given that the consumption will be increasing over time.

Similarly as we analyzed in the case of increasing in government spending, the impact of an increase in e will decrease the real price of bonds. It follows from equation (3.3), and knowing $\overline{m}' > \overline{m}$, clearly the initial jump of q must be negative. So on the impact, q instantaneously drop to the level less than unity. Figure (2.9) depicts the co-movement of q and bonds reserves b:

The real price of bonds, q, declines on impact implied by equation (2.25) and then increases over time as indicated by the phase diagram (2.9). So in this case, the financial rate s depreciates proportionally less than the official rate. As long as q remains below unity, the individual has an incentive to overinvoice in exports and underinvoice in imports, thus causing a decrease in the stock of bonds. This will continue until q surpass the unity and overshoot the long run steady state, which induces an underinvoice in exports and overinvoice in imports, thus re accumulates the bonds reserves back to $\overline{b}$. Figure (2.10) and (2.11) illustrate the dynamic path "m-q" sub-space and in "m-b" sub-space. Both the schedules move rightwards after the devaluation.

Similarly as we analyzed in the last case, the real interest rate r. rises on impact and drops down over time.

It is clear that from equation (2.16) and (2.19), q will remains at unity across the steady states and the bonds reserve will remain at constant $\overline{b}$ across steady states, too. Intuitively, when the devaluation in domestic currency occurred, a rise in the real money balance implies that the nominal money balance will increase. Meanwhile, the consumption drops down. The economy runs a current account surplus during the adjustment. Individuals will sell bonds to smooth their consumption, which lowers the real price q down below unity. As long as q is less than one, the presence of leakage would causes individuals to overinvoice in export and underinvoice in import, which in turn, causes a decreasing of bonds reserve. In this process exporters will sell bonds to acquire foreign currency, which in term trade in with domestic money. Henceforth, cause a decrease on the stock of bonds. Once q decreases to the extent, which undershoots its long-run value of unity, it will induce underinvoice in export and overinvoice on import. This is the driven force in the incomplete duel exchange market to drive the economy back to equilibrium. Real interest rate r jumps up wards as a natural consequence to the change of q. Later on, when individual starts buying foreign bonds at a attempt to secure the value of their wealth when inflation became alleviate, r reduces monotonically to induce a upward consumption path.

(C) Change in the stock of domestic credit

The other common and important disturbance we would like to address here is an unanticipated and permanent decrease in the stock of domestic credit. This exogenous disturbance causes similar consequences as we have seen in the case of devaluation. A critical difference here lies on the difference response of the price level. By the similar analysis, we may see that on the impact, the price level decreases instead of jump up as is in the case of devaluation. The real money jumps up on the impact and then is adjusted across the transitional interval to a new steady state. The new steady state of real money $\overline{m}'$ is greater than the original level $\overline{m}$. But whether m will increases across the steady states depends on the initial jump of the price level. The adjustment path of the real price and the real interest rate of bonds is similar as in the case of devaluation. Thus induced by the change of real price q, the reserve of bond b and the consumption c are supposed to have the similar behavior as we discussed before.

We summarize the results obtained in this section:

(1) The bonds reserve appears cyclical scenario generated by fraudulent leakage. This result is consistent with Flood and Vitale’s study in reserve and exchange rate cycles

(1998).

(2) The existence of fraudulent leakage reduced the disturbances by exogenous impacts, both from government expenditure and from currency devaluation. It is the leakage based on cross-market transactions that engage in the performance to maintain the unstable system around its steady state level.

(3) Dual exchange rate regime is an unstable system. In the long run, unification is an ultimate destiny.

2.4 Unification of Dual Exchange Rates

The discussion in last sections shows that dual exchange rate system is nevertheless an unstable system. In order to achieve long run equilibrium, countries with dual exchange markets must transfer to a unified exchange rate system. Consequently, a country adopting dual track system will not be able to maintain in stable permanently. The historical records have shown that many countries that adopted dual exchange rate system have sooner or later transferred into some kind of unified exchange rate system.

In this section we will study the transition from a dual regime to a unified exchange rate regime. First we determine the solution of the exchange rate addressed by the model in section (2.2). By using the assumption of zero speculative profit, we can solve the solution explicitly. Second, we analyze the alternatives targets of transition from dual regime to a unified regime. We will end this section by a discussion about the volatility prior to the transition caused by non-market fundamentals.

(a) The exchange rate solution

From the model in section (2), the definition of $q=s/\varepsilon$, equation (2.7) and the assumption that the official rate "ε" follows the crawling peg exchange rate regime, we can obtain the general solution of the exchange rate for the dual exchange rate region: (See appendix (II.4) for the derivation)

$$s(t) = -c_1 \frac{r}{r^*} e^{pt} + c_2 e^{pt - r^* t} \tag{4.1}$$

Where c_1 and c_2 are two constants to be determined.

Prior to the transition, the authority, as well as the market will set exchange rates at levels such that speculators will be failed to anticipate speculative profits by entering the market at the moment right before the transition.

As having been discussed by R P. Flood and N. P. Marion (1974), given no information what exchange rate regime will be adopted after transition, speculators can only estimate the expected value of each foreign exchange based on the available information up to date, if the transition will be made to that particular regime. The absence of expected speculative profits requires the expected unified exchange rate will be

$$x(T\_) = p_1(t)V_1(T) + p_2(t)V_2(T) + \cdots + (1 - p_1(t) - p_2(t) - \cdots - p_{k-1}(t))V_k(T) \tag{4.2}$$

Where p_i, i=1,2,3…k-1, is the subjective probability imposed by speculators at time t to an actual transition to a specific exchange rate regime "i", and the $V_i\ (T)$ s are the expected value of a particular exchange rate regime at time T, if the actual transition is made to this regime. T stands for the time when the transition occurs, while T- is the time right before the transition. This expression states that the unified exchange rate just before the transition will be a weighted average of foreign exchange under each possible exchange regime.

Together with the initial condition set at the time t = 0, the condition that speculators abandon expected profits prior to the transition date will form the terminal condition to allow us to determine the value of c_1 and c_2 in the solution of (4.1).

At time 0, the time prior the adoption of dual regime, the exchange rate is

$$s(0) = c_1 \frac{r}{r^*} + c_2 \tag{4.2}$$

The absence of expected speculative profit at time T implies

$$s(T) = -c_1 \frac{r}{r^*} e^{pT} + c_2 e^{pT - r^*T}$$
$$= p_1(t)V_1(T) + p_2(t)V_2(T) + \cdots + (1 - p_1(t) - p_2(t) - \cdots - p_{k-1}(t))V_k(T) \tag{4.3}$$

Combining (4.2) with (4.3), we can solve the equations to determine:

$$\bar{c}_1 = \frac{s_0 e^{(p-r^*)T} - G(p_i(t), V_i(T))}{\frac{p}{r^*} e^{pT}(1 + e^{-r^*T})}$$
$$\bar{c}_2 = s_0 - \frac{s_0 e^{(p-r^*)T} - G(p_i(t), V_i(T))}{e^{pT}(1 + e^{-r^*T})} \tag{4.4}$$

where $G(p_i(t), V_i(T))$ is the weighted value of all possible exchange regimes. This completes the solution of the dynamic path of the unified exchange rate. Now we can specify the dynamic path of the unified exchange rate:

$$s(t) = -\frac{r(s_0 e^{(p-r^*)T} - G(p_i(t), V_i(T)) e^{p(t-T)}}{p(1 + e^{-r^*T})} + (s_0 - \frac{s_0 e^{(p-r^*)T} - G(p(t), V(T))}{e^{pT}(1 + e^{-r^*T})}) e^{(p-r^*)t} \tag{4.5}$$

For a simple example, if there are only two alternatives of targeted foreign exchange regimes: fixed regime and floating regime, and the corresponding initial and terminal values are 6.5 yuan per dollar and 8.2 yuan per dollar respectively, the unified exchange rate will be

$$s(t) = -c_1 \frac{r}{r^*} e^{pt} + c_2 e^{pt - r^*t} \tag{4.6}$$

where

$$c_1 = \frac{s_0 e^{(p-r^*)T} - \{p(t)6.5 + (1 - p(t))8.2\}}{\frac{p}{r^*} e^{pT}(1 + e^{-r^*T})}$$

$$c_2 = s_0 - \frac{s_0 e^{(p-r^*)T} - \{ p(t)6.5 + (1 - p(t))8.2\}}{e^{pT}(1 + e^{-r^*T})} \tag{4.7}$$

Figure (2. 13) depicts the simplest case of foreign exchange right before the crisis. The above discussion states that the new exchange rate prior to the transition will be a weighted average of official rate and the floating financial rate in order to drive the speculative profit to zero.

(B) Alternative targets of exchange rate unification

Based on the model developed in section two, we will examine the transition process from a dual regime to alternative target regimes and the corresponding consequences. While economic agents are uncertain about the nature of the post-transition regime, it is the economic authority's major task to determine what will be the target regime after the transition. Among all the considerations, a smooth process of the transition is always a major concern for authorities. We will see that it might take longer to generate a smoother transition to a target regime. If the authority needs to complete the transition quickly, the cost will be volatility in the exchange market.

Two cases will be discussed. (1) The target is a floating exchange rate regime. (2) Targeted on a crawling peg regime. Although fixed regime will be another alternative, since there is almost no room for absolute pegged regime after the crush of Bretton Woods system in 1972, crawling peg regime provides a wider range of generality.

(1) Crawling peg regime.

Solution (2.4.1) describes a floating dynamic path of exchange rate during the dual regime. During the dual regime period, there also exists a commercial exchange rate following a crawling peg regime with the depreciate rate π set by central authority. These two dynamic paths are not overlapped in general. For a smooth transition, central authority may choose a crawling peg exchange rate as the post transition target. First, by adjusting the rate of depreciation, the central authority can gradually reduce the difference between the commercial rate and the financial rate, therefore to generate a smooth transition. Second, if the time is allowed, the central authority can adjust the rate π, so that the exchange rate value of the financial rate and the value of the commercial rate will be identical at the terminal point of time, thus to complete the transition at a pre-planned time T.

This can be described by the following equation. Suppose at time T, the financial rate and the commercial rate are identical, that is:

$$s(T) = -\bar{c}_1 \frac{r}{r^*} e^{pT} + \bar{c}_2 e^{pT - r^* T} = e_0 e^{pT} = e(T) \tag{4.8}$$

The left side is the value of floating financial regime at time T. The right side is the value of crawling peg regime at time. Together with equation (4.7) and (4.8), we have:

$$\frac{-s_0 e^{(p-r^*)T} - s_0 e^{(p-2r^*)T} + G(1+e^{-r^*T})}{e^{pT}(1+e^{-r^*T})} = e_0 + s_0 e^{-r^*T} \tag{4.9}$$

where the G is a weighted average of expected foreign exchange rate values at time T. From this equation we can literally solve for T in terms of the depreciation rate π:

$$\hat{T} = \frac{\ln G - \ln s_0(1+e_0)}{p - r^*} \tag{4.10}$$

It is apparently that when the rate of depreciation π increases, the time of transition will be decreased, therefore to complete the transition sooner. Together with the accommodation of adjustment of other macro economic fundamentals, crawling peg regime provides a policy choice that the adjustment of a single instrument "π" can significantly alter the length of transition. This would be an advantage for authority to choose crawling peg as a target regime.

By adjusting the rate π, the central authority can precisely determine the time of transition. Since the financial rate and the commercial rate are adjusted to approach each other, the transition will be a smooth one.

But since the step for either crawling peg or the adjusting in rate π must be small, this transition will not be easy to complete in a relatively short period of time. However, the historical cases in Italy, France and other western European countries show that most of the transitions did not take for long. It usually completes in an extraordinary short period of time. The most recent transition in China showed some kind of similarities as in those countries during seventy's: It was planned to complete in a smooth but longer term scope, but it turned out to be forced to complete in a very short period of time. And most transition ended up with a managed floating regime. We here discuss the second case:

(2) Floating exchange regime.

In (4.1), the dynamic solution of exchange rate is determined by the terminal condition, which is the weighted mean of all possible value of targeted exchange regimes at time T. If the alternate target is floating regime, once the actual regime switch was announced and set at time T, there will be a discrete jump to the expected or real floating regime value. When agent is uncertain of the nature of the post- regime transition, it is unavoidable for this kind of discrete jump. This will cause volatility during transition or even after transition, but it does not need a slow movement to complete the transition. The transition could be completed instantly, thus to avoid the possible speculative attack prior to the transition. In real world, once economic authority decides to transfer from dual regime to target regime, very often the development of situation will be beyond their control. Authorities are willing to see a gradual, smooth transition so that they can use all instruments to ensure the economy in a regulated track. For economic agents, they aim at the possibility of speculative profit, or at least protect themselves in the transition process. Based on the available information, expectations made by agents would be likely to overreact over policy signals, hence to cause market volatility. Authority would be enforced to speed up the transition either for the purpose to protect foreign reserve or stabilize monetary policy to prevent possible inflation, or to defend external financial attack. We have seen that authorities switched their transitional strategies targeted from crawling peg to floating regime. The interesting results is, during transition from dual exchange rate regime to unified exchange rate regime, volatility are not caused by economic fundamentals or policy inconsistency, but often by individual agent's expectation and the uncertainty factors in the process. We depict the situation in Figure (2.13).

(C) Volatility prior transition and its subjective probability

As to the volatility during and before the transition time, there is one aspect of the confusion in exchange market, which has rarely explained in literature. What caused the exchange rate volatility prior to the transition? Since economic authorities proposed on a stable transition, monetary and fiscal policies are applied to ensure that macroeconomic fundamentals will be stable. However, we have barely seen any country ever experienced a stable process prior transition. According to most of the research, dynamic path of exchange rate should depends on market fundamentals such as foreign reserves, money supplies, government spending, current and capital account balances and so on. But reality has shown some kind of inconsistency with theories. The volatility in exchange rate path did not in coherence with market fundamentals. Our model here provides another explanation. (4.1) shows that, except for the money growth rate π, the dynamic path of exchange rate does not depend on most of those fundamental elements. Rather it is affected by the subjective probabilities settled by economic agents. Because of the incomplete information and temporary nature of transition, agents make their subjective expectation of the probabilities towards the future target regime only based on temporary and frequently changed information. Many of them are likely to be rumors but could dramatically stir psychological disturbances. As long as the transition is not completed yet, agents have to adjust their prospective expectations. As a major factor in the exchange rate path, (4.5) and (4.6) demonstrates that the change in $G(p_i(t), V_i(T))$ will affect the exchange path, hence to cause the volatility prior to the transition. In addition to the affects from market fundamentals, agent's subjective probability of the prospective regime and the corresponding value of each exchange regime play even more important role in the movement in exchange rate prior to transition.

2.5 Conclusions

This paper applies an optimization framework to establish an incomplete segment dual exchange rate model. For the majority of multi-variable systems described by differential equations, normally we cannot literally solve the solutions from the system simultaneously. Therefore systems such as these are very difficult to analyze. However, by linearize the differential equations, knowing that some of the information will be lost, we obtain a fourth order linear differential equation system generated from the model. This differential equation system enables us to analyze a dynamic co-movement of four important variables, namely real money, real foreign asset and real price as well as real interest of the foreign bond.

We use this model to examine the effects of exogenous disturbances on the system. Unexpected disturbances such as expansionary fiscal policy, devaluation of the domestic exchange rate and a change in the domestic credit have been considered. Unlike most papers dealing with cross-market transaction leakage, we found that the presence of leakage buffers the exogenous shocks by reducing the magnitude of the jump in some variables caused by the indigenous disturbances on the system. Instead, fraudulent leakage leads to a kind of oscillatory and cyclical movement in the dynamic path of these variables. The reason for such a movement is because the presence of leakage. Any effect obtained from the differential between the official and market exchange rates will be offset by an opposite effect, which is generated in the same process as well. In this way the leakage phenomenon brings a free market mechanism into a dual exchange market, which is originally regulated largely by government administration force, and consequently reduces the volatility caused by exogenous disturbances.

The purpose for the adoption of dual exchange system is to control capital flows to prevent balance-of-payments pressure (Cumby,Robert E. 1984). This model implies that the government might tolerate the existence of leakage in order to reduce the impact from exogenous disturbances. While keep loosing the control on capital flow, the fraudulent leakage does helps the economy to remain around its steady states. Is this a trade off? For the case of China, the disadvantage of dual system has been criticized from various aspects. Efficiency and social welfare losses caused by the fraudulent leakage related to the large differentials between the official rate and the market rate yields a big question about the viability of the system. However. the dual system existed for more than decade. Does this give us some implication?

With the tendency that the difference between the two-exchange rate disappears over time, it implies that a dual exchange rate regime can not sustain in the long run. A study of the transition from the dual system to a unified exchange rate system, especially the dynamic transitional process is an attracting topic. The main advantage of dynamic approach is to illustrate the movement of a system by the corresponding dynamic paths. However, it is totally depending upon the solution from the system, mostly, solution from differential equation system. However, we successfully solved the dynamic path of floating exchange rate. This provides a convenient vehicle to discuss the unification of dual exchange rate regimes.

As we are aware that using the linearizing approach to solve differential equation system often leads to a loss of information. Methodological improvement including the introducing numerical methods seems a great challenge to economists working on the dynamic problems.

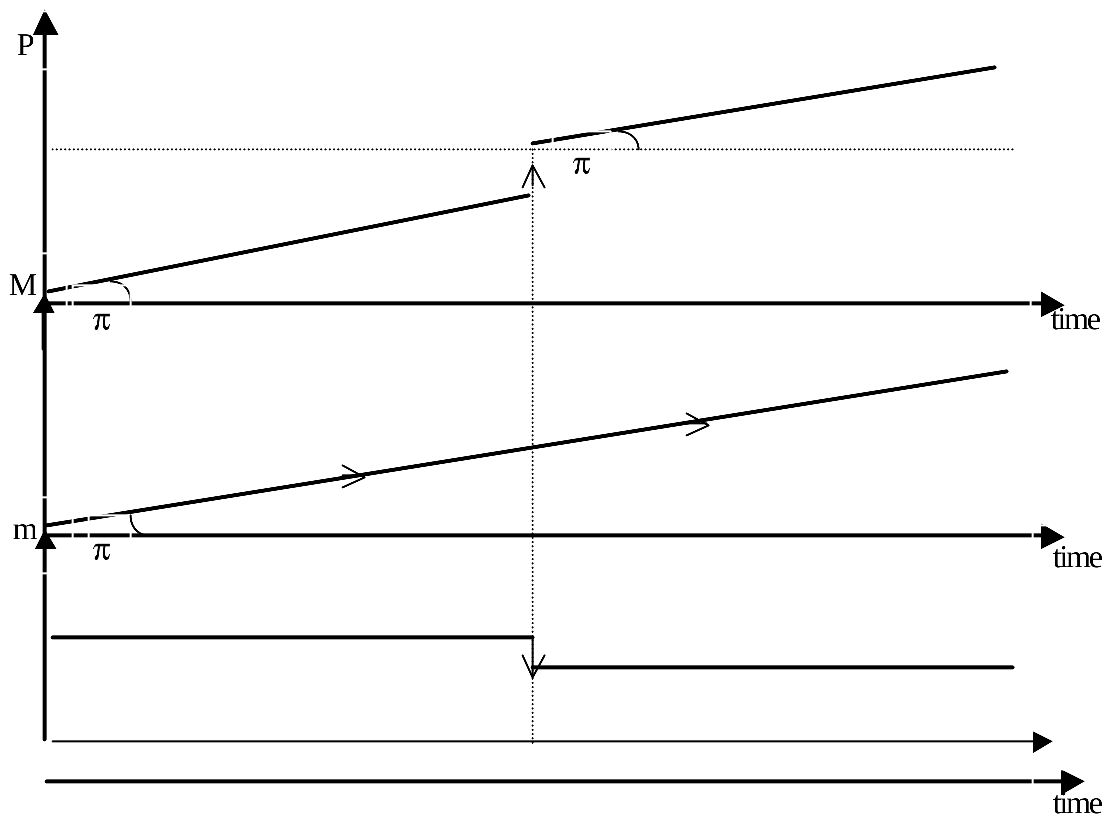


Figure (2.1) Dynamics of Nominal Money,

Real Money and Price Jump up

Phase Diagrams on the Effect of Unanticipated Increase in Government Spending (without Fraudulent Leakage)

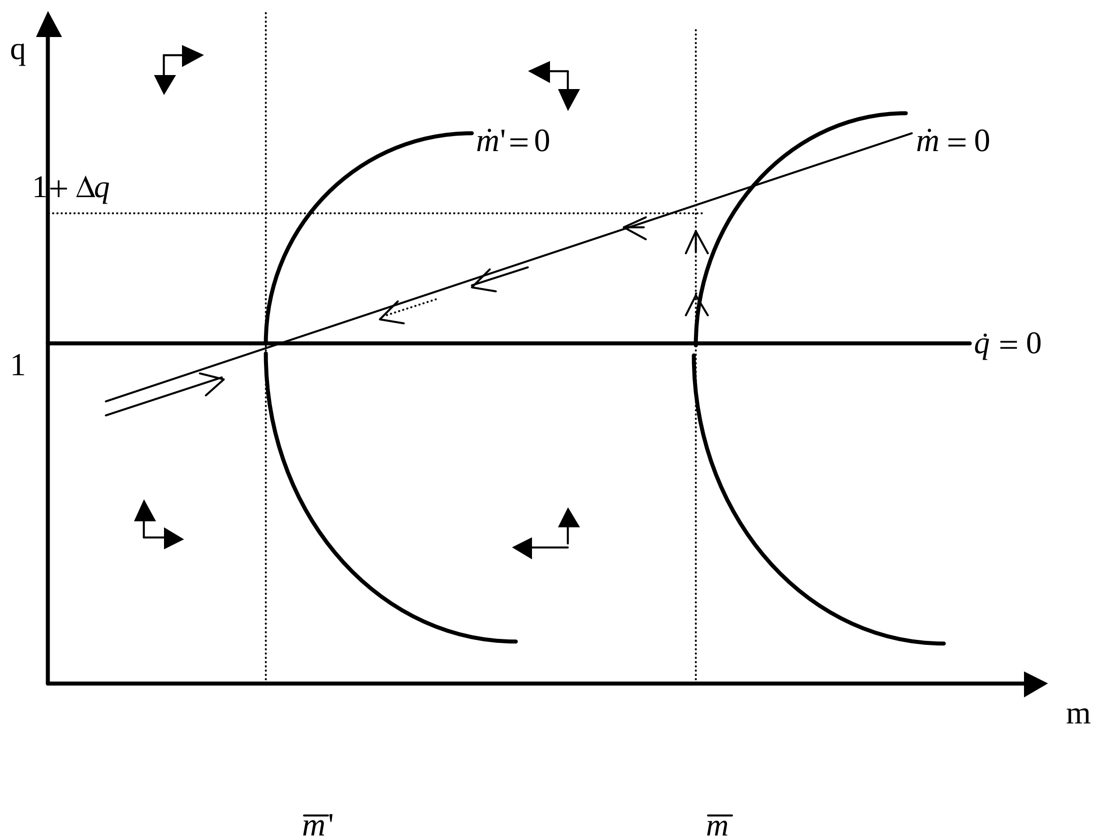


Figure (2.2) Phase Diagram of Real Bonds Price

Phase Diagrams on the Impact of Unanticipated Increase in Government Spending (with Fraudulent Leakage)

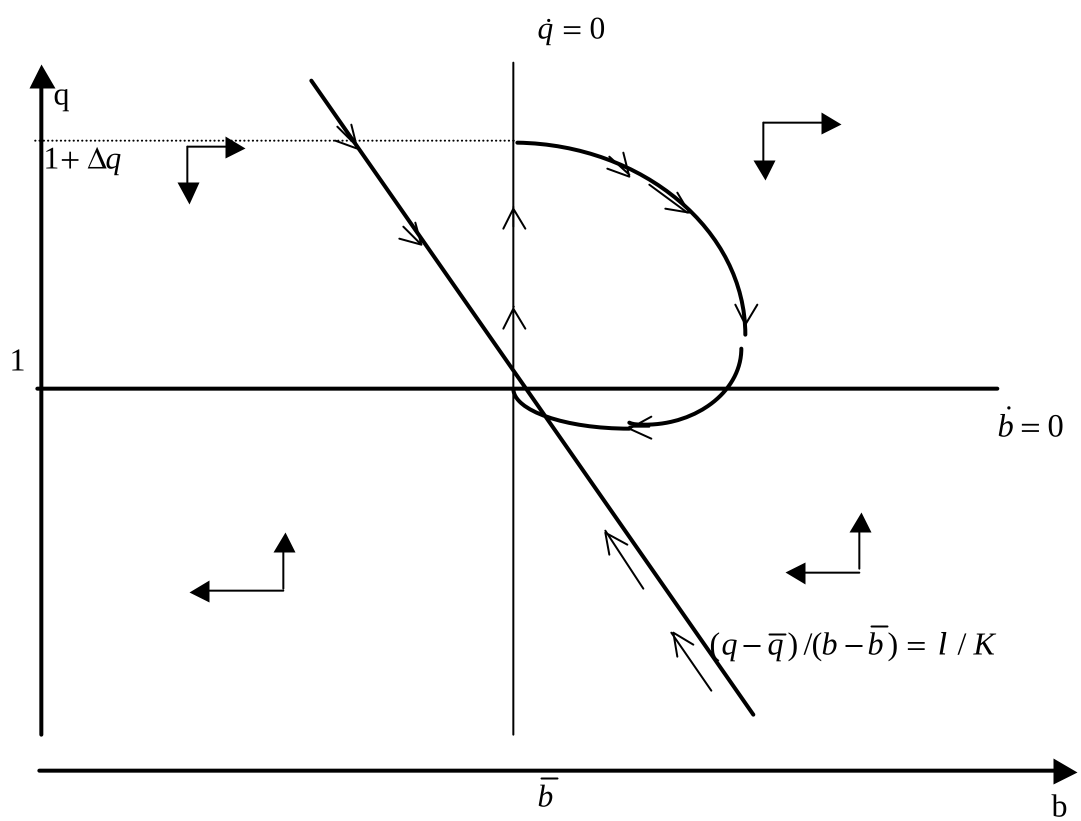


Figure (2.3) Phase Diagram in Real Price of Bonds and Bonds Space

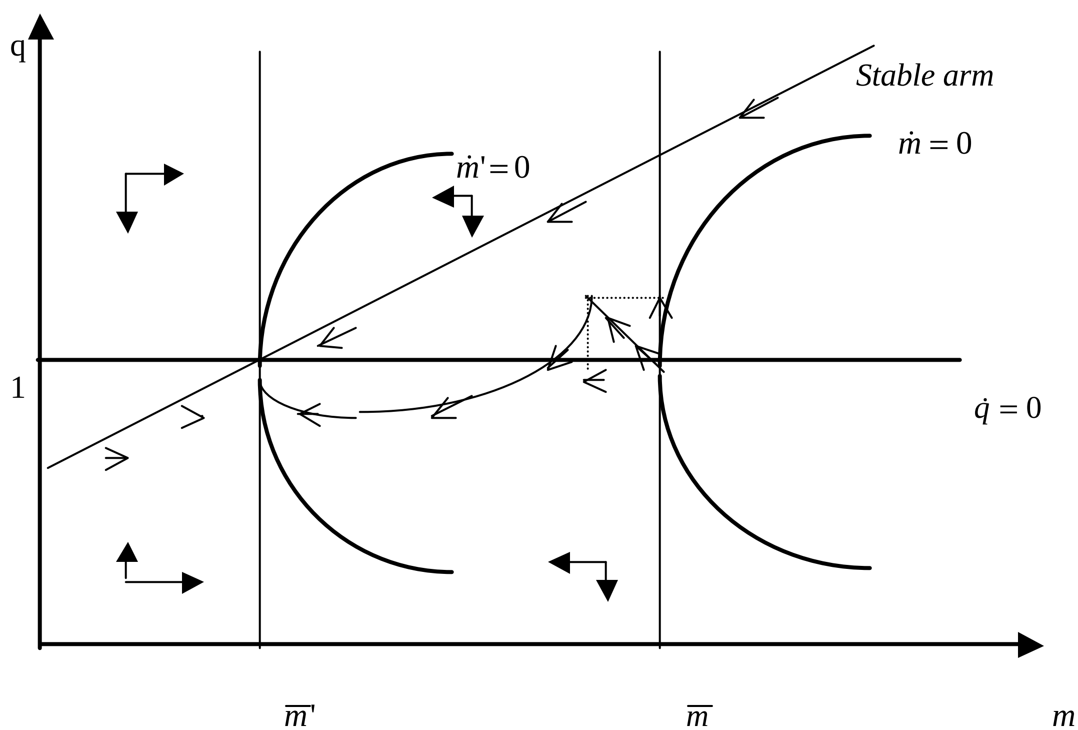


Figure (2.4) Phase Diagram in

Real Bonds Price and Money Space

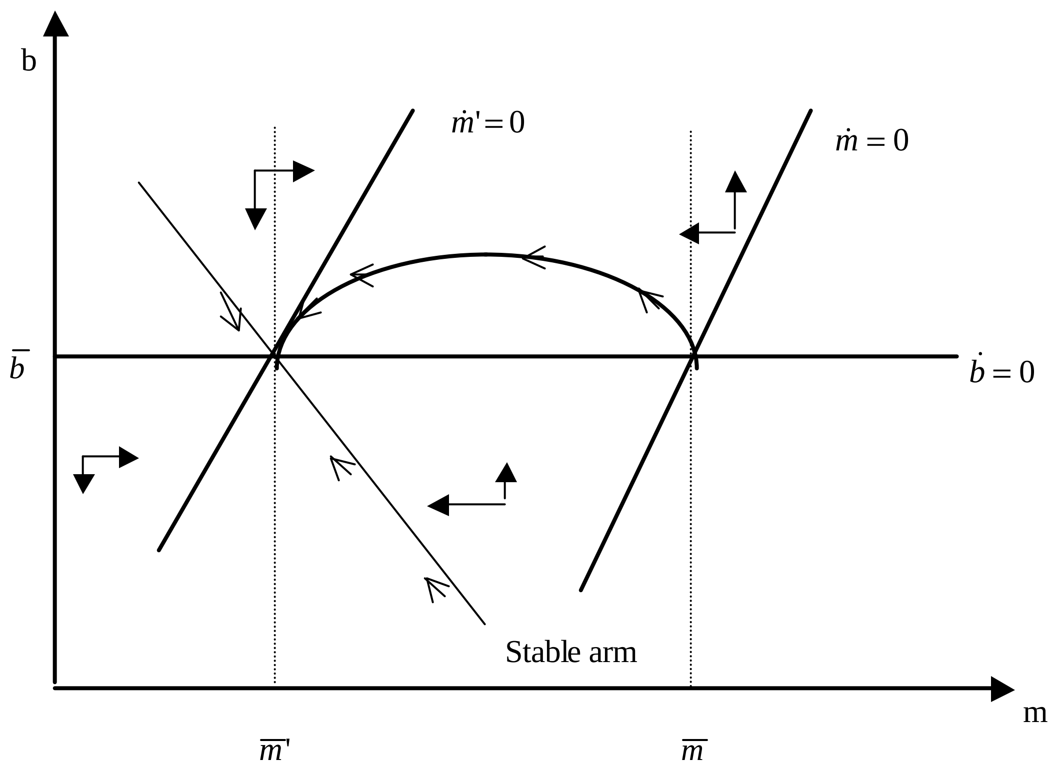
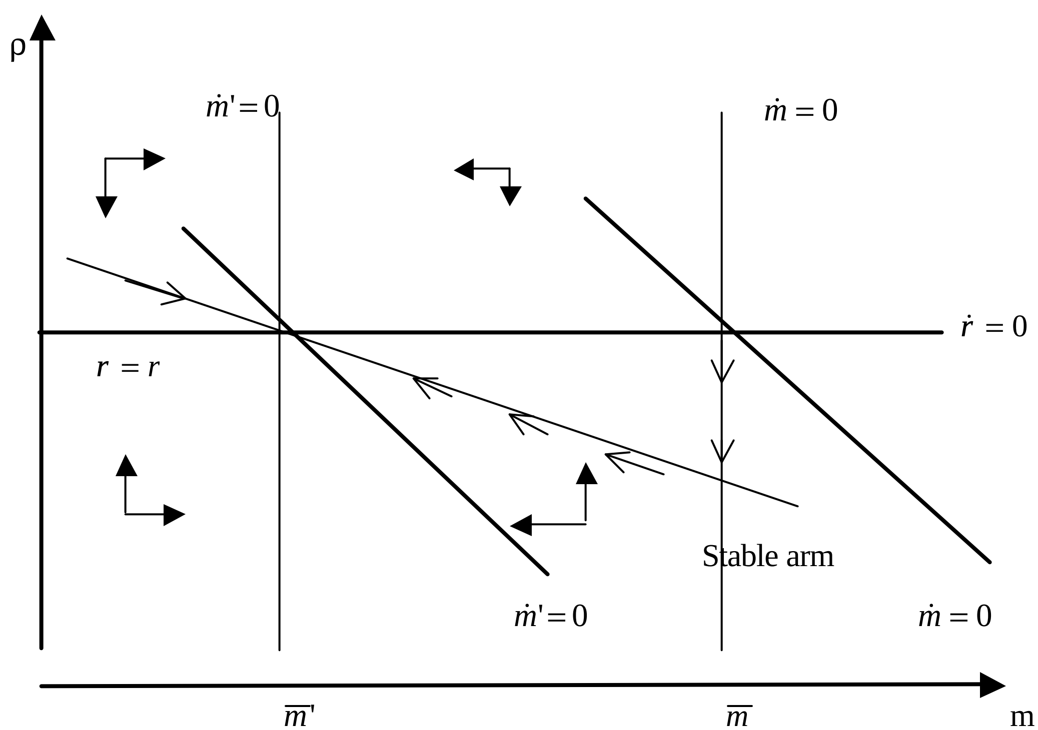
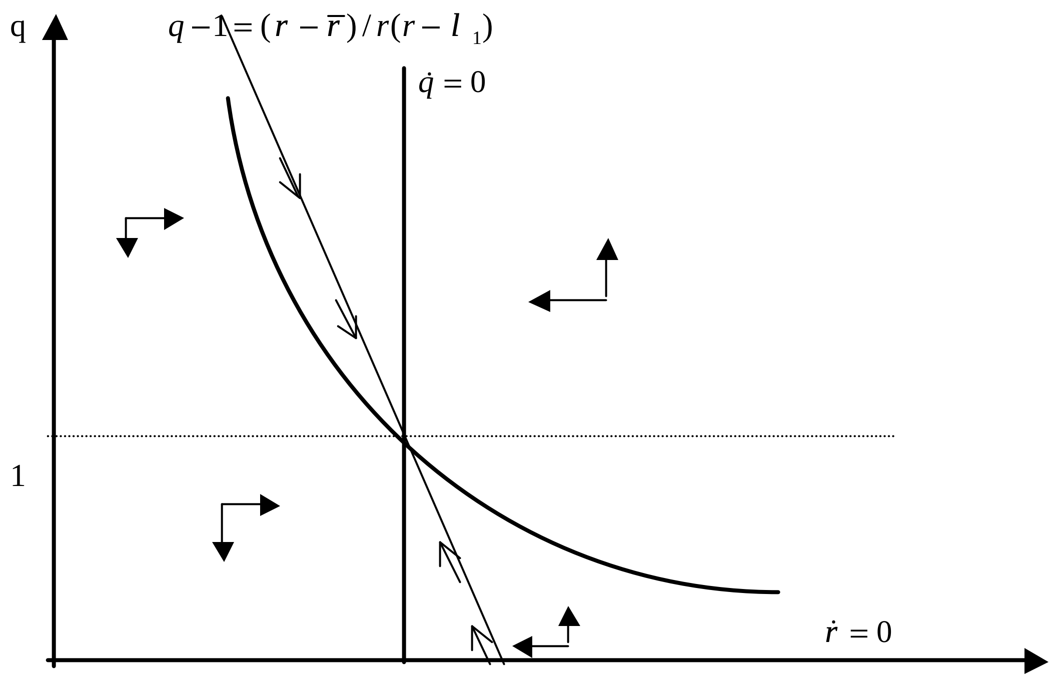


Figure (2.5) Phase Diagram of Real Bonds and Money



ρ
$\dot{m}'=0$
$\dot{m}=0$
$\dot{r}=0$
$r=r$
Stable arm
$\dot{m}'=0$
$\dot{m}=0$
$\overline{m}'$
$\overline{m}$
m

Figure (2.6) Phase Diagram of Real Interest Rate and Money



r ρ

Figure (2.7) Phase Diagram of Exchange Rate ratio and Real Bonds Price

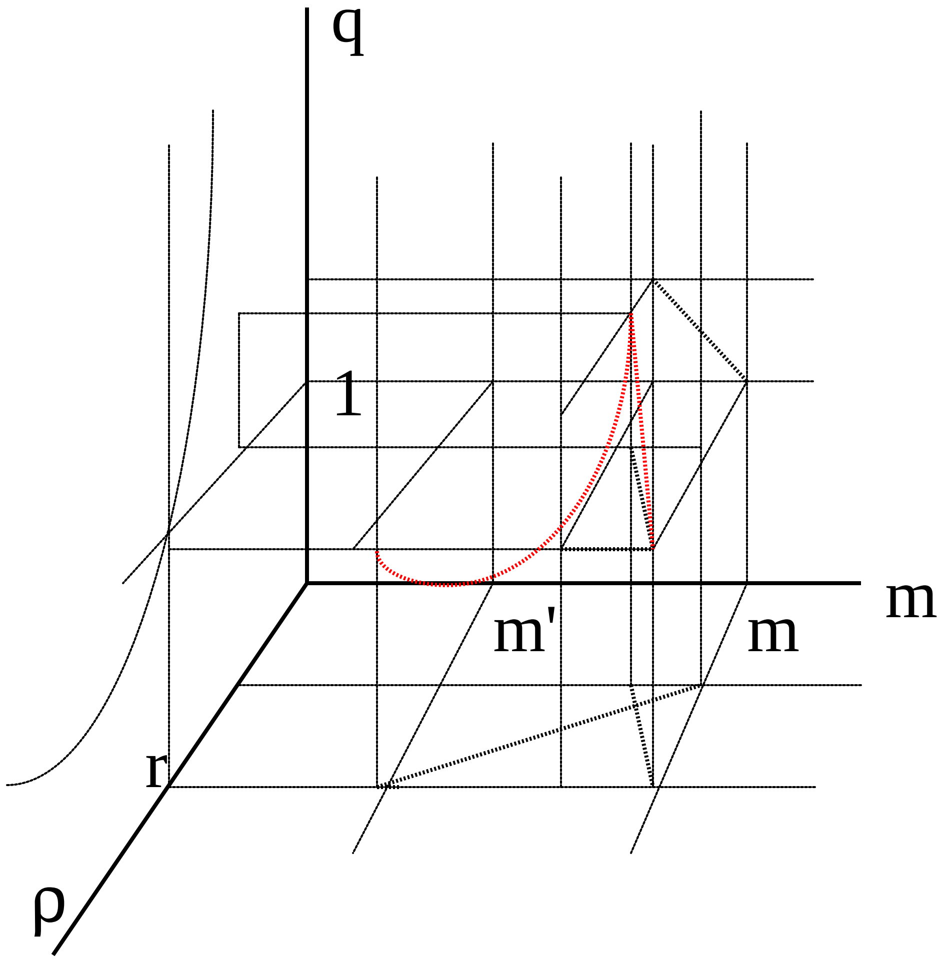


Figure (2.8) Dynamic Movements in a Three Dimensional Space

Phase Diagrams on the Impact of Unanticipated Currency Devaluation

(with Fraudulent Leakage)

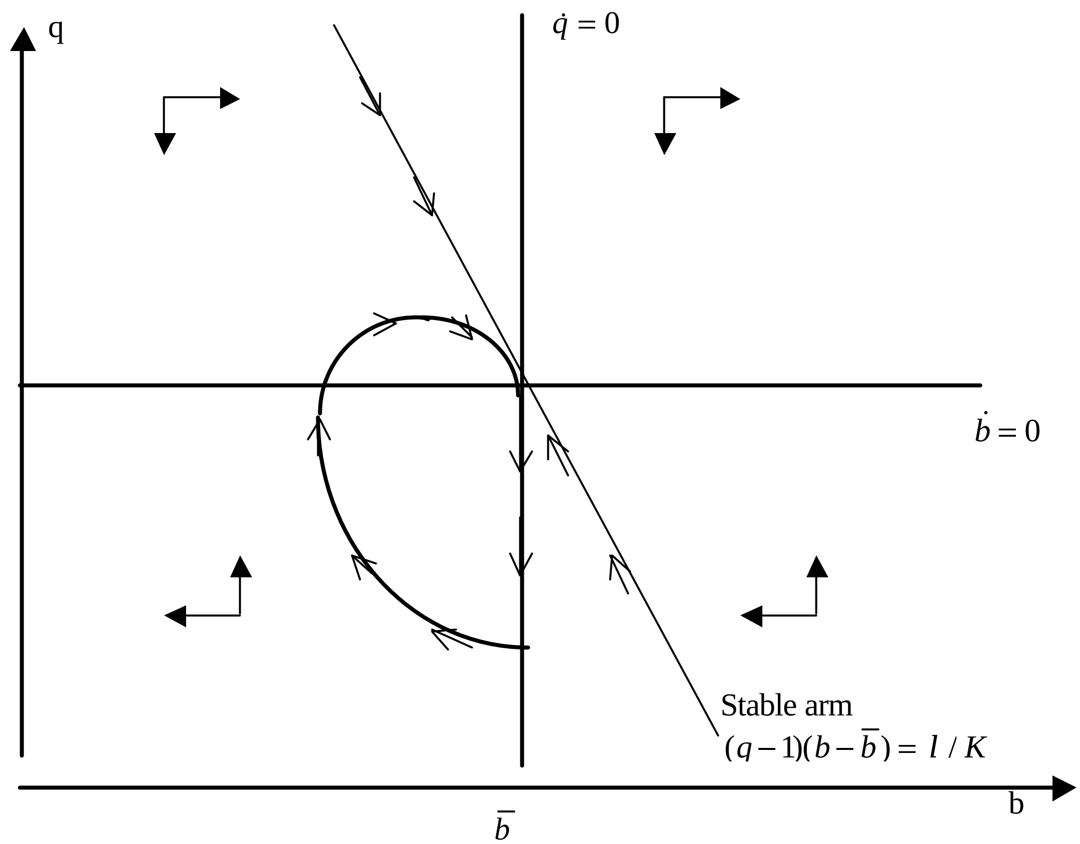


Figure (2.9) Phase Diagram in

Real Price of Bonds and Bonds Space

(With Leakage)

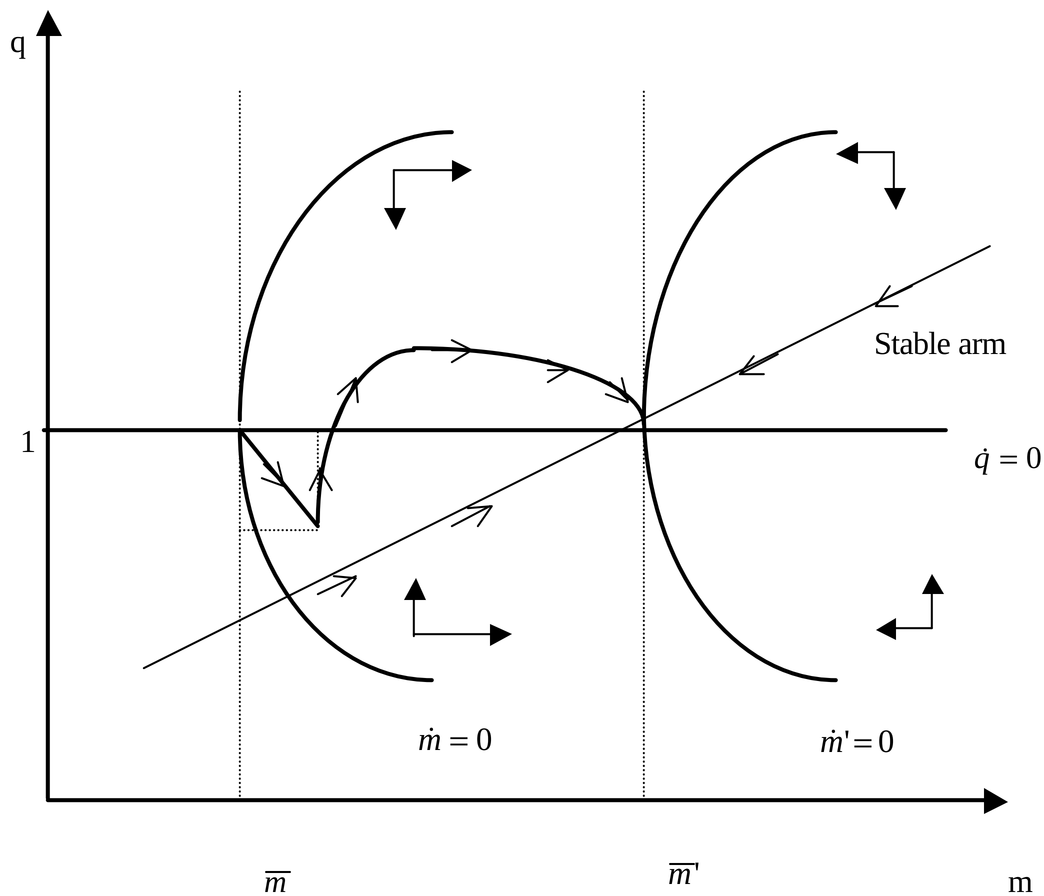


Figure (2.10) Phase Diagram in

Real Bonds Price and Money Space

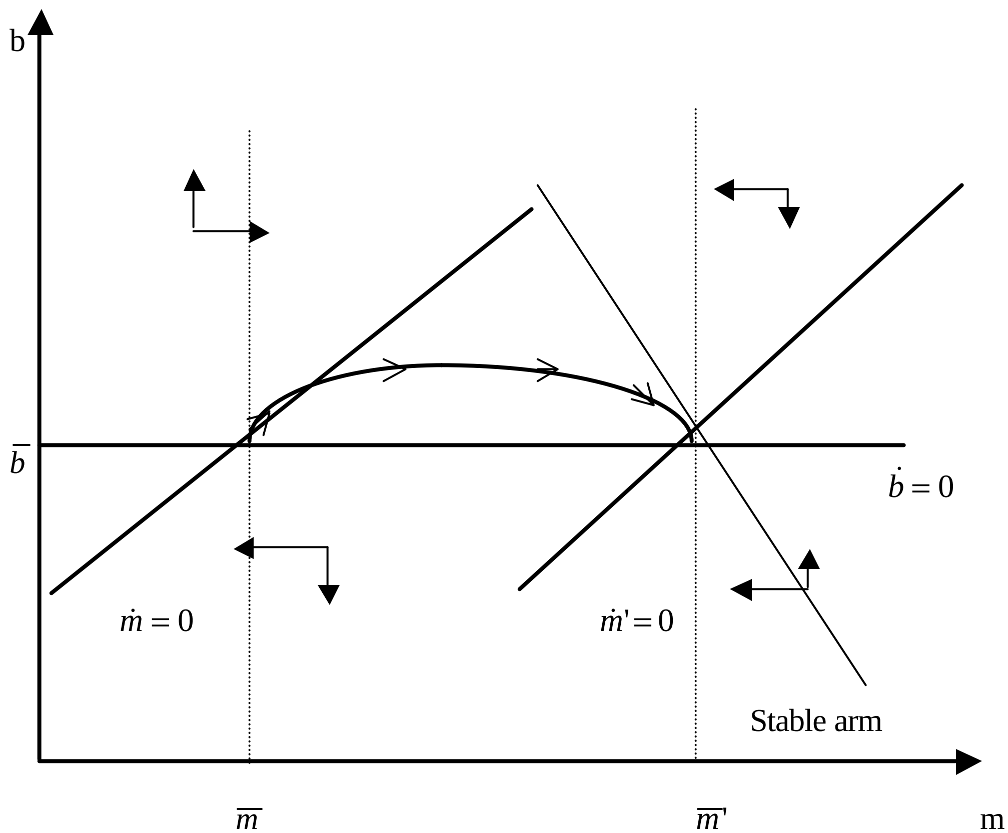


Figure (2.11) Phase Diagram of Real Bonds and Money

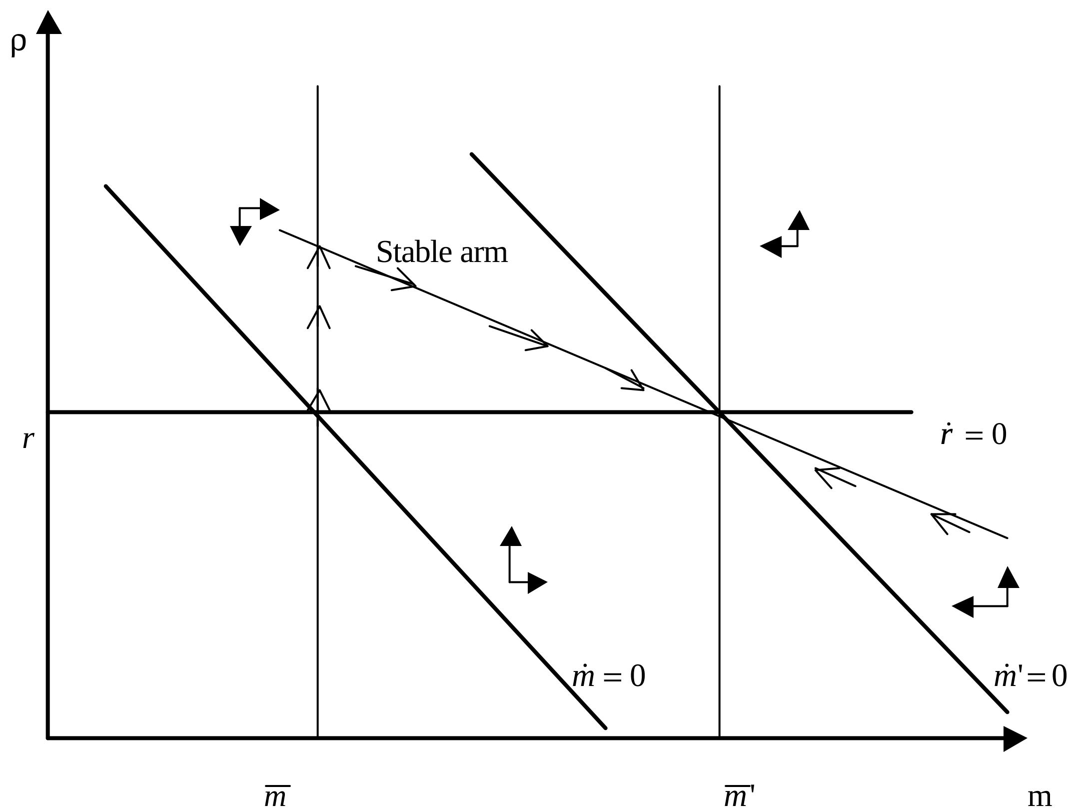


Figure (2.12) Phase Diagram of Real Interest Rate and Money

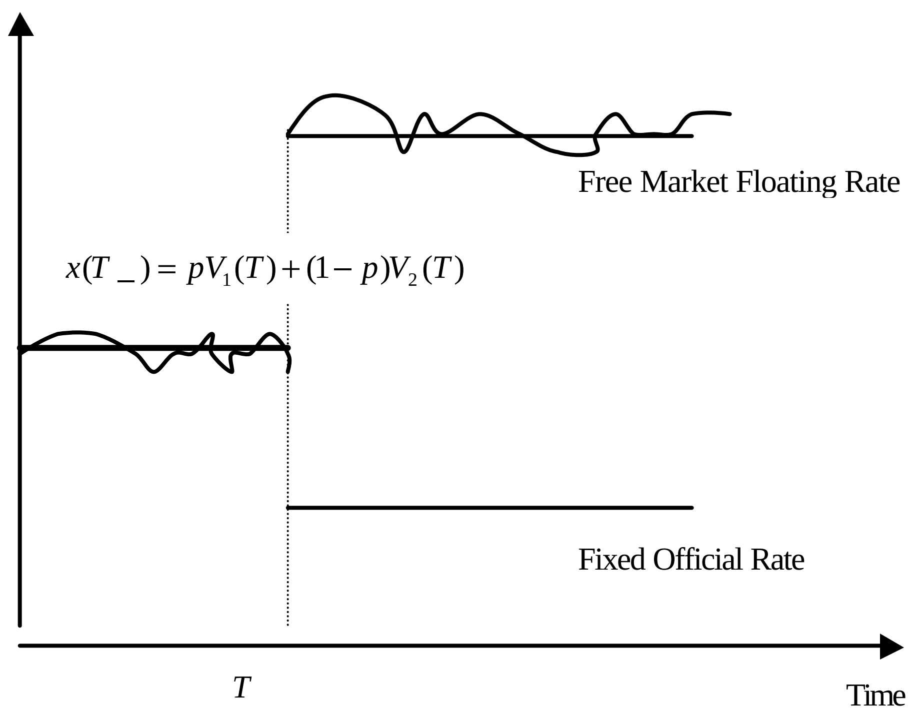
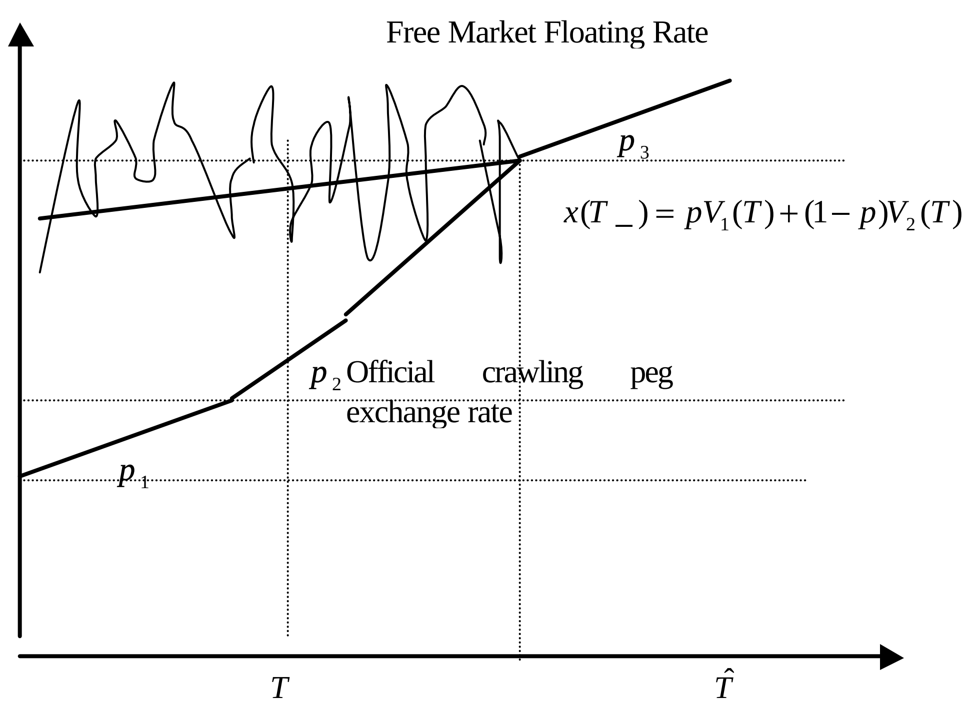

Free Market Floating Rate
$x(T\_) = pV_1(T) + (1-p)V_2(T)$
Fixed Official Rate
T
Time

Figure (2.13) Exchange Rate Unification

without speculative profit



Time

Figure (2.13) Exchange Rate Unification

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APPENDIX II:

APPENDIX AND SOLUTIONS TO CHANPTER (2)

<u>Appendix (II.1) The matrix (A) in Chapter (2) has only one negative igenvalue</u>

For the matrix (A) of (2.20) in section two, the eigenvalues of A are solved from the characteristic polynomial P(λ) generated from the characteristic equation $\left| A - lI \right| = 0$. Polynomial P(λ) can specified as:

$$P(l) = l^4 - (A + B + r)\, l^3 + (AB + rB - rA)\, l^2 - (rAB + Kr\frac{U_{cc}}{a^2 \overline{l}})\, l \qquad (\text{A.1})$$

where A = (r - 1/α - π) < 0, B = (r + 2π + 1/α) > 0.

For the equation P(λ) = 0 there will be four roots, namely l_{1_1}, l_2, l_3, l_4. According to the Weida theorem, which describes the relationship between coefficients and roots, a 4'Th order polynomial P(λ) can be described as

$$P(l) = l^4 - (\sum_{i=1}^{4} l_i)\, l^3 + (\sum_{i,j=1, i\neq j}^{4} l_i l_j)\, l^2 - (\sum_{i,j,k=1,\ i\neq j\neq k}^{4} l_i l_j l_k)\, l + l_1 l_2 l_3 l_4 \qquad (\text{A.2})$$

the coefficient of the third order term, namely the coefficient of term l^3 is :

$$l_1 + l_2 + l_3 + l_4$$

which is equal to the trace of (A), equal 3r+π > 0.

meanwhile, the coefficient of the term of λ is less than zero, Specifically this coefficient is equal to

$$l_1 l_2 l_3 + l_1 l_2 l_4 + l_1 l_3 l_4 + l_2 l_3 l_4 < 0. \qquad (\text{A.3})$$

Assuming that $l_4 = 0$, then (A.3) becomes $l_1 l_2 l_3 < 0$.

Now it is clear that there is at least one negative eigenvalue.

Since the trace of A is greater than zero and the product of the three eigenvalue is less than zero, therefore, there is only one negative eigenvalue. Without loosing generality, we can denote that

$l_1 < 0, \; l_2 > 0, \; l_3 > 0, \; l_4 = 0.$

This proves the conclusion in section two.

Appendix (II.2) To prove that the first eigonvalue $l_1 < r - 1/a - p$.

We prove here that the location of the unique negative eigenvalue l_1 must be in the area of λ < r - 1/α - π. In appendix (I) we showed there is only one negative eigenvalue. Let's check the value of P(r - 1/α - π), where

P( ) is the characteristic polynomial.

$$P(r-\frac{1}{a}-p)=\left|A-(r-\frac{1}{a}-p)I\right|$$

$$=\begin{vmatrix} 0 & 0 & 0 & r \\ \Phi(r-\frac{1}{a}-p) & 3p+\frac{2}{a} & 0 & \Phi r \\ 0 & 1 & \frac{1}{a}+p & 0 \\ 0 & 0 & K & -r+\frac{1}{a}+p \end{vmatrix}$$

$$=r\Phi(r-\frac{1}{a}-p)^2<0$$

Since P(0) = 0 and P(λ) is monotonically increasing in the area r -1/α - π <λ < 0. Thus the negative root can only be locate in the area λ < r - 1/α - π.

Appendix (II.3)   To prove an increase in government spending will cause inflation

From section (II), we have the following equations, which describes the resource constraint, the government budget constraint and the money supply:

$$\begin{cases} y = c + g \\ t = rf + mp - g \\ m = f + D/p \end{cases} \tag{A.3.1}$$

Using the cash in advance constraint and the first equation, we have

$$m = a(y - g) \tag{A.3.2}$$

(A.3.2) together with the second equation of (A.3.1) derives

$$m = \frac{t + rD/p + g}{r + p} \tag{A.3.3}$$

equating (A.3.3) with the third equation of (A.3.1), we can express the price p in terms of g, the government spending:

$$\frac{rD}{p} = a(y - g)(r + p) - t - g \tag{A.3.4}$$

It is easy to see

$$\frac{dp}{dg} = \frac{p^2(1 + a(r + p))}{rD} > 0.$$

So an increase in government spending will cause inflation.