External Shocks and Collapsing the Pegged Exchange Rate System†

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Abstract

In this paper, we build a monetary model that focuses on external factors that cause the collapse of the pegged exchange rate system. It is shown that the pegged exchange rate system cannot survive under a persistent external shock, if this shock dominates all other offsetting factors. Given the large scale of the shock, the peg will collapse even if the domestic policy and the fundamental are sound. The policy options for the government under this circumstance depend on the nature of the shock. Capital control is effective only if the shock is capita outflow shock. A current account deficit shock, however, requires more capital inflow to prevent the depletion of foreign reserves and therefore requests for less capital control. Another important contribution of this paper is to distinguish the effect of capital mobility with that of the asset substitutability. We show that these two effects can be totally different on the pegged exchange rate system under different shocks.

Keywords: the pegged exchange rate system, external shock, currency crisis.

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1. Introduction

A common feature of the existing currency crisis models is that researches in the literature consider only domestic economic weakness, in terms of either the financial system or government policy as the root cause of currency crisis (see, for example, Krugman (1979), Flood and Garber (1984), Obstfeld (1996), Chang and Velasco (2001), and Tsang and Ma (2002)). Although this characteristic fits the history in some countries, we do not believe it has explained all of the causes of currency crises.

This paper builds an alternative monetary model under such an economic environment that there is no persistent fiscal deficit that requires the expansion of domestic credit. Also there is no obvious trade-off between domestic macroeconomic target and the exchange rate stability that the Central Bank has to choose. The domestic financial system has the both stable credibility and vulnerability. In brief, the domestic economy does not display any visible domestic cause, or accumulated domestic causes, of crisis.

However, the pegged exchange rate regime of this economy still has to be given up merely due to the large-scale external, negative shocks to the capital account or the current account continuously. The domestic currency has to depreciate eventually.

Under this circumstance, the government must first judge whether attacks are from the capital account or the current account, if they attempt to strengthen controls through capital inflow or outflow to maintain official-parity of the exchange rate. If attacks come from the capital account, then strengthening the control of capital outflow may postpone the collapse of the parity. However, if they increase the capital
control to deal with the current account shock, then it will be counterproductive and the opposite result will occur, i.e., it will accelerate the collapse of the parity.

In this paper a monetary model is constructed specifically to demonstrate this idea. The topic is especially important for China. After the open door policy adopted in 1978 and the accession to the WTO in 2001, China has been facing the challenge of how to maintain the stability (Ma and Tsang, 1997) and at the same time to enhance the efficiency of her domestic financial system (Zhao, et al, 2002; Ma, et al, 2003; Li and Ma, 1996). This challenge becomes more acute after China opened up her domestic securities markets to foreign investors under the newly adopted system of qualified foreign institutional investors (QFII) in Dec. 2002 (CSRC, 2002) and has been considered the system of qualified domestic institutional investors (QDII) seriously recently. These are two transitional schemes to liberalize China's capital account when the Chinese currency is not fully convertible (Liu, et al, 2002). At the same time, Chinese economic and financial development and its stability are increasingly important to the world economy (Ma, 2001a; Hu and Ma, 1999; Ma, et al, 1998).

The remainder of the paper is organized as follows. Section 2 presents a monetary model structure and its solution. Section 3 analyzes how the major parameter changes in this model may alleviate the currency crisis occurrence. Finally, Section 4 concludes.

2. Model structure and its solution
First we have the standard the money demand function (Ma and Kanas, 2000; Sun, 2000):

$$\frac{M^d(t)}{P(t)} = \alpha_0 Y - \alpha i(t)$$  \hspace{1cm} (1)

where Y represents the real income. Similar to other currency crisis model, Y is assumed to be constant. \(i(t)\) is domestic nominal interest rate at time \(t\). \(M^d(t)\) is nominal money demand. \(\alpha_0\) and \(\alpha\) are parameters. To simplify our analysis, we also assume that price index \(P(t)=1\) is a constant.

Money supply is defined as:

$$M^s(t) = \lambda [D + R(t)]$$  \hspace{1cm} (2)

\(M^s(t)\) is nominal money supply at time \(t\). \(D\) is domestic credit created by the central bank. The assumption for \(D\) in our model is different from that in the first generation model of currency crisis, which usually assumes that \(D\) grows along with the fiscal deficit. In this paper we assume \(D\) is constant, which demonstrates the disconnection between monetary policy and the budget deficit. \(R(t)\) is the official foreign exchange reserves (denominated in domestic currency). \(\lambda\) is the money multiplier.

When domestic money market is in equilibrium: \(M^d(t) = M^s(t)\), we have:

$$i(t) = \frac{\alpha_0 Y - \lambda D - \lambda R(t)}{\alpha}$$  \hspace{1cm} (3)

The ‘desired’ net demand level for the stock of domestic assets is (see, for example, Ma, 2001b; and Ma, et al, 2002):

$$K^*(t) = s[i(t) - i^* - \hat{e}(t)] + v_k t$$  \hspace{1cm} (4)

where \(i^*\) is foreign nominal interest rate, which is assumed to be constant for
simplicity. \( \dot{e}(t) \) is the expected change of nominal exchange rate \( e(t) \). If agents believe the pegged exchange rate regime can be maintained, then we have \( \dot{e}(t)=0 \).

The premium paid to domestic assets over foreign assets is defined as \( i - i^* - \dot{e} \). And \( s \) is a parameter that reflects the degree of substitution between domestic and foreign assets. If \( s = +\infty \), investors are indifferent between holding domestic assets and foreign assets. It is the case of perfect substitutes. If the return on domestic assets would exceed the foreign assets, then demand for domestic assets would be infinite that would reduce the differential of the returns. This implies under perfect substitution assumption, the uncovered interest parity must hold: \( i - i^* - \dot{e}=0 \).\(^1\) If \( s = 0 \), investors take the view that domestic and foreign assets are completely not substitutable. This implies that domestic assets demand does not depend upon international yield differentials, \( i - i^* - \dot{e} \), at all. If \( 0 < s < +\infty \), domestic assets and the foreign assets are imperfect substitutes.

The exogenous capital account shocks, \( v_k \), is completely driven by exogenous factors. \( v_k t \) is accumulated shocks up to time \( t \). \( K^*(t) \) is the ‘desired’ level of the stock of domestic assets.

Equation (4) implies that the net demand for domestic assets depends on both economic fundamentals as well as other exogenous international factors. For example, Japan entered a long-term decline and deflation period since burst of the bubble economy in the early 1990s. The interest rates move back and forth for a long time near the zero level. The economy has fallen into "the Japanese type liquidity trap".

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\(^1\) Perfect asset substitution occurs only if there is perfect capital mobility. We do not consider the rare case that assets are perfect substitutes under capital control.
Although the interest rates are usually higher in the host countries of the Japanese foreign investment, such as East Asian countries, than the interest rate in Japan, there has been a persistent capital outflow back to Japan from these countries due to the financial difficulties experienced in Japan herself. So has $v_k < 0$.

Suppose $K(t)$ is the actual net demand level for the stock of domestic assets. If $K(t)$ is not at the level of $K^*(t)$, then there will be capital inflow or outflow to adjust the level of $K(t)$ until it is reached to the same level as $K^*(t)$. Define the capital inflow to be allowed to at any time $t$ is:

$$dK(t)/dt = c \left[ K^*(t) - K(t) \right]$$

$$= c s \left[ i(t) - i^* - \dot{\epsilon}(t) \right] + c v_k t - c K(t), \quad 0 \leq c \leq +\infty,$$

where $c$ is capital adjustment coefficient which represents the capital mobility. To certain extent, the authority controls the capital mobility. If $c = +\infty$, it is perfect capital mobility and there is no capital control. This implies investors are able to adjust their investment portfolio to the desired level instantaneously. If $c = 0$, it means the capital is under complete control and there is no capital mobility, even though $K(t)$ is different with $K^*(t)$. If $0 < c < +\infty$, it indicates the capital is under partial control and the capital mobility is imperfect. In each of the adjustment instant, the quantity of the capital stock adjustment is only a portion of the difference between $K(t)$ and $K^*(t)$.

For simplicity, we assume that the economic system is at steady state when there is no shock. Namely the initial values are:

$$K(0)=K^*(0)=0, \quad i_0 = i^*, \quad i_0 = \left[ \alpha_0 Y - \lambda D - \lambda R_0 \right] / \alpha$$

where $i_0$ is initial domestic nominal interest rate for $t=0$. $R_0$ is the initial foreign
exchange reserve level.

In our model, we have differentiated the two concepts of international asset substitution and international capital mobility. This distinction was initiated by Dornbusch and Krugman (1976) but has not been well taken into consideration formally in the existing literature. For example, Fleming (1962), Mundell (1963), Clarida, Gali and Gertler (2002) combined the two concepts of instantaneous portfolio adjustment and perfect asset substitutability into perfect capital mobility. Frenkel and Rodriguez (1982), and Mark (2001) incorporated slow adjustment of portfolio and imperfect asset substitutability into one single concept of imperfect capital mobility.

However, in our analysis, it is important and significant to distinguish the two concepts of international asset substitution and international capital mobility. Theoretically, asset substitutability will affect the composition of international assets in the portfolio at the ‘desired’ level, whilst the capital mobility determines how much capital stock can be held at any time actually. It will affect the feasibility, costs, and time lag of the capital stock adjustment. In practice, the asset substitutability is mainly determined by the discrepancies of macroeconomic environments and financial systems between countries. It is unlikely to be changed dramatically in the short run. However, the capital mobility is mainly restricted by the foreign exchange regulations of governments, in addition to the constraint of the international settlement facilities. As government regulations may change in the short run, capital mobility may be affected immediately.

For an open economy, changes in the foreign exchange reserve of the central bank
\( dR(t)/dt \) is determined by changes in international capital stock \( dK(t)/dt \) and the current account balance \( CA \):

\[
dR(t)/dt = dK(t)/dt + CA 
\]

(6)

Changes in the foreign exchange reserve of the central bank \( dR(t)/dt \) therefore also is the balance of payment. Under the floating exchange rate system without the central bank intervention, \( dR(t)/dt=0 \). Under the pegged exchange rate system, \( dR(t)/dt \) may have various possible values. For simplicity, we assume the current account remains a constant.

Integrating (6), we may obtain the stock level of the foreign exchange reserve of the central bank:

\[
R(t) = R_0 + K(t) + CA \ t
\]

(7)

where \( R_0 \) is the initial level of foreign exchange reserve.

From (5), (4) and (3), together with the assumption that investors believe that the pegged exchange rate regime is held, i.e., \( \dot{e}(t) = 0 \), we obtain:

\[
\frac{dK(t)}{dt} = \phi S \left[ \frac{\alpha_0 \lambda D - \lambda R(t)}{\lambda} - \lambda i \right] + \nu K(t) - cK(t) 
\]

(8)

Substituting (7) and (3) into (8), we obtain

\[
\frac{dK(t)}{dt} = \phi S \left[ \frac{\alpha_0 \lambda D - \lambda R_0 - \lambda K(t) - \lambda CA \ t}{\lambda} - \lambda i \right] + \nu K(t) - cK(t) 
\]

\[
\phi S \left[ -\lambda \frac{K(t) - \lambda CA \ t}{\lambda} \right] + \nu K(t) - cK(t) 
\]

(9)

Hence
\[ \frac{dK(t)}{dt} + \beta_2 K(t) = -\beta_3 t \]  

(10)

where

\[ \beta_2 = cs\lambda/\alpha + c > 0 \]

\[ \beta_3 = c[s\lambda CA/\alpha - v_k] \]

The solution of (10) is:

\[ K(t) = (1 - e^{-\beta_3 t}) \frac{\beta_2}{\beta_2^2} - t \frac{\beta_3}{\beta_2} \]  

(11)

Taking derivative of (11) with respect to time \( t \):

\[ \frac{dK(t)}{dt} = -t(1 - e^{-\beta_3 t}) \frac{\beta_2}{\beta_2^2}, \]  

(12)

Substituting (12) into (6), we find that the balance of payment is a function of the net capital inflow, the current account balance \( CA \), international capital substitutability \( s \), and capital mobility \( c \):

\[ \frac{dR(t)}{dt} = \frac{dK(t)}{dt} + CA = -t(1 - e^{-\beta_3 t}) \frac{\beta_2}{\beta_2^2} + CA \]  

(13)

3. The implications of the model solution: occurrence of the crisis and its postponement

Rewrite (13) as follows:

\[ \frac{dR(t)}{dt} = CA \left[ 1 - (1 - e^{-\beta_3 t}) \frac{s\lambda/\alpha}{1 + s\lambda/\alpha} \right] + v_k \frac{1 - e^{-\beta_3 t}}{1 + s\lambda/\alpha} \]

\[ = CA \frac{1 + e^{-\beta_3 t} s\lambda/\alpha}{1 + s\lambda/\alpha} + \frac{\lambda^2}{v_k} \frac{1 - e^{-\beta_3 t}}{1 + s\lambda/\alpha}, \]  

(13c)

To simplify notation, define the balance of payment as:
\[ \Delta(t) \equiv \frac{dR(t)}{dt}, \quad (13b) \]

From (13c), it can be shown that if there is capital account outflow due to the exogenous shock \( v_k < 0 \), and current account deficit \( CA < 0 \), then we have:

\[ \Delta(t) < 0. \quad (13a) \]

From (3) and (5), we can identify two coordinating adjustment mechanism of the interest rate in our model. The first one is internal adjustment mechanism. From (3) it is shown that if the foreign exchange reserve falls and results in domestic currency supply to drop, the interest rate \( i(t) \) will rise to constrain domestic money demand. This will maintain domestic money supply and demand balanced. On the other hand, equation (5) shows that the rise in domestic interest rate will attract the foreign capital inflow. Thus foreign exchange reserve is augmented that alleviates the pressure of exchange rate devaluation.

Nevertheless, equation (13a) shows that the effect of the interest rate adjustment mechanism is limited. Under a persistent negative shock to the balance of payment, the foreign exchange reserve will fall continuously. It will be depleted completely at some point of time in the future. Then the pegged exchange rate will collapse.

This indicates that even the domestic credit supply is sound, if there is an external shock either \( v_k < 0 \) or \( CA < 0 \), the domestic interest rate adjustment mechanism itself is not sufficient to defend the pegged exchange rate system. Although the interest rate adjustments may help the foreign reserve correction to offset the impact of capital outflow on the exchange rate, their adjustments are limited and therefore cannot eliminate the deficits in the balance of payments. As a result, official foreign reserve
will have to continuously fall and the exchange rate parity will eventually be abandoned.

Substituting (11) into (7), we also find that the official foreign reserve \( R(t) \) is a function of its own initial level \( R_0 \), the exogenous attacks to the capital account \((v_k)\) and to the current account \((CA)\), the international capital substitutability \((s)\), and capital mobility \((c)\):

\[
R(t) = R_0 - e^{-\beta t} \left[ \frac{\beta_3}{\beta_2^2} - \frac{\beta_3}{\beta_2} + \frac{\beta_3}{\beta_2^2} + CA \right] \tag{14a}
\]

Suppose the time of complete depletion of official foreign exchange reserve, namely the life of the pegged exchange rate system, is \( t_N \), \( R(t_N)=0 \), when the balance of payment continues to be in deficits. We have:

\[
R_0 - e^{-\beta t_N} \left[ \frac{\beta_3}{\beta_2^2} + \frac{\beta_3}{\beta_2} - (\frac{\beta_3}{\beta_2} - CA) \right] t_N = 0 \tag{14}
\]

Obviously, \( t_N \) is an implicit function of initial level of foreign reserve \( R_0 \), the exogenous attacks to the capital account \((v_k)\) and to the current account \((CA)\), the international capital substitutability \((s)\), and capital mobility \((c)\). By (14), we can find out all the derivatives of \( t_N \) with respect to all these variables. Thus we may analyze the effects of these variables on the \( t_N \).

### 3.1. The impact of initial foreign exchange reserve \( R_0 \)

According to (14) and (14c), we have

\[
\frac{dt_N}{dR_0} = \frac{1}{\left(1 - e^{-\beta t_N} \left[ \frac{\beta_3}{\beta_2^2} - CA \right] \right)} = \frac{-1}{\Delta(t_N)} \tag{15}
\]
where $t_N$ is the time of devaluation occurs.

Assume that the balance of payments continue to be in deficits. This is the situation of the most of the currency crises under which the default risk of the foreign payment is rising. From (13a), when the balance of payments continues to be in deficits, $\Delta(t_N) < 0$, hence $\frac{d.t_N}{dR_0} > 0$. This means an increase in the initial value of foreign reserve may lengthen the life of the pegged exchange rate. This conclusion is consistent with the first generation currency crisis model.

3.2. The impact of the current account shock

According to (14) and (14c), we have

$$\frac{dt_N}{dCA} = t_N \left(1 - \frac{s\lambda}{s\lambda + \alpha}\right) + (1 - e^{-\beta t_N}) \frac{s\lambda}{\alpha(s\lambda/\alpha + 1)^2} - \Delta(t_N)$$

(16)

where $t_N$ represents the time of devaluation occurs.

When the balance of payments continues to be in deficits, $\Delta(t_N) < 0$, hence

$$\frac{dt_N}{dCA} > 0$$

This shows that a reduction of the current account deficits (namely $CA$ increases) may delay the depletion of the reserve. Therefore it is advantageous to sustain the pegged exchange rate system.

3.3. The impact of the capital account shock

From (14) and (14c), we obtain:
\[
\frac{dt_N}{dv_k} = -\frac{c}{\beta_2} \left[ t_N - \frac{1}{\beta_2} (1 - e^{-\beta_2 v}) \right] \Delta(t_N)
\]

where \( t_N \) represents the time of devaluation occurs.

Define the function in the bracket of the numerator as \( f(t) = t - \frac{1}{\beta_2} (1 - e^{-\beta_2 v}) \), with \( f(0) = 0 \) and \( df(t)/dt = 1 - e^{-\beta_2 v} > 0 (t>0) \), namely \( f(t) \) is a positive monotonic increasing function with respect to \( t \). Hence, \( [t_N - \frac{1}{\beta_2} (1 - e^{-\beta_2 v})] > 0 \).

When the balance of payments continues to be in deficits, we have \( \Delta(t_N) < 0 \), \( \frac{dt_N}{dv_k} > 0 \). This implies that an increase of the negative shocks to the capital account (namely the \( v_k \) decreases) will accelerate the collapse of the pegged exchange rate system.

In conclusion, the pegged exchange rate system still faces the total destruction under persistent external shocks even when the fundamentals of an economy are strong and domestic economic policy also is sound. This conclusion shows that the pegged exchange rate system is intrinsically vulnerable. However, despite its weakness, our subsequent analysis shows there are policies that may prolong the life of the pegged exchange rate to certain extent, although not permanently.

3.4. The impact of capital mobility

From (14) and (14c), we obtain:
\[
\frac{dt_N}{dc} = \frac{\beta_3 \left( 1 - e^{-\beta_3 t_N} - t_N e^{-\beta_3 t_N} \right)}{c \beta_2 \Delta(t_N)}
\]  

(18)

where \( t_N \) represents the time of devaluation occurs.

Define the function in the bracket of the numerator as \( g(t) = \frac{1 - e^{-\beta_3 t}}{\beta_2} - t e^{-\beta_3 t} \),

with \( g(0) = 0 \), and \( \frac{dg(t)}{dt} = t \beta_2 e^{-\beta_3 t} > 0 \) \((t > 0)\), namely \( g(t) \) is a positive monotonic increasing function with respect to \( t \). Hence, \( \left[ \frac{1 - e^{-\beta_3 t}}{\beta_2} - t e^{-\beta_3 t} \right] > 0 \).

We find that the impact of capital mobility \( c \) on the time of depletion of official foreign reserve is dependent upon the source of the shock to the balance of payments. There are two cases in consideration.

3.4.1. There are only the capital outflows due to the capital account shock, \( v_k < 0 \). But there is no current account shock, \( CA = 0 \).

Hence, \( \beta_3 = - c v_k > 0 \), \( \Delta(t_N) < 0 \), and \( \frac{dt_N}{dc} < 0 \).

This indicates that if the root cause of currency crisis is the capital outflow due to the capital account shock, then the strengthening of capital control by the government may enhance the robustness of the pegged exchange rate to the exogenous shocks.

Malaysian government may provide a good example of the effectiveness of this policy during the Asian financial crisis. Another example is the Chinese currency Renminbi exchange rate which remained unaffected by the Asian financial crisis. The non-liberalization of the capital account in China is part of the reason behind the success of Renminbi.
3.4.2. There are only current account deficits, \( CA < 0 \). But there is no capital account shock \( v_k = 0 \).

Then, we have \( \beta_3 = c s \lambda \frac{CA}{\alpha} < 0 \), and \( \Delta(t_N) < 0 \). Hence we have \( \frac{dt_N}{dc} > 0 \).

This indicates that if the root cause of currency crisis is the current account deficit, then the strengthening of capital control by the government may, however, hinder the capital inflow and thus weaken the pegged exchange rate system.

From the results of Sections 3.4.1 and 3.4.2, we know that if the balance of payments continues to be in deficits, then we must identify clearly whether it is the current account deficit or the capital outflow shock. The strategy of the foreign exchange control is completely opposite in the two situations.

3.5. The impact of the degree assets substitution

From (14) and (14c), we have:

\[
\frac{dt_N}{ds} = -\Delta(t_N)
\]

where \( t_N \) is the time of devaluation occurs.

From Theorem 1 in the Appendix of this paper, we know that no matter it is the capital account outflow shock \( v_k < 0 \), or the current account deficit shock \( CA < 0 \), we always have

\[
\Delta(t_N) < 0 \text{ and } \frac{dt_N}{ds} > 0.
\]

This indicates that if the balance of payments continues to be in deficits, an
increase in the assets substitution degree $s$ may delay the foreign reserve depletion. It is advantageous to the pegged exchange rate parity.

The degree of assets substitution $s$ is mainly determined by the heterogeneity between domestic and foreign financial systems and the macroeconomic environments. For instance, with the liberalization of the domestic financial market and stable economic development in a developing country such as China (Ma, 2001a), substitution between domestic and foreign assets will probably increase. Based on our model’s analysis, this will be advantageous to the stability of the exchange rate system.

To conclude, it is found that the effects of asset substitution and capital mobility have both similarities and differences. As far as our knowledge, this is a new contribution to the literature. If the root of a currency crisis is the current account deficit, then enhances the capital international mobility and increases the asset substitution degree may maintain the exchange rate parity. Nevertheless, if the cause of a currency crisis is the capital outflow due to the exogenous shock to the capital account, then the policy response should be completely opposite, i.e., the government should strengthen the control of capital outflow and try to increase the degree of asset substitution. This distinction also is our paper’s important contribution to this area of research.

4. Conclusion

Through a currency model with asset substitution and imperfect capital mobility,
This paper shows that the pegged exchange rate may not be able to survive under external shocks that are large enough and their duration is sufficiently long. Thus the currency crisis will eventually occur, even if domestic fundamentals are strong and domestic policy is sound and consistent with the exchange rate parity. The policy implications from our analysis are as follows.

If external attacks are from the capital account, then strengthening the capital control is helpful to maintain the pegged exchange rate regime. If external attacks are from the current account, then the authorities should relax the capital control that is the correct measure to maintain the pegged exchange rate regime.

For example, as China is steadily to liberalize her capital account, our conclusion is especially important. It suggests that if future negative shock to the Chinese balance of payment is from current account, then the increase of capital mobility and due to the capital account liberalization will stabilize the Renminbi exchange rate. However, if future negative shock to the Chinese balance of payment is from the capital account, then enhanced capital mobility followed by the capital account liberalization will destabilize the Renminbi exchange rate.

Moreover, another important finding of this paper is that there are similarities and differences between asset substitution and capital mobility for the impacts on the exchange rate parity. If the balance of payments continues to be in deficits, an increase in asset substitution may delay the reserve depletion and is helpful to the pegged exchange rate system. The previous researches that embed the portfolio adjustment speed and the asset substitution in the concept of capital mobility
obviously have their limitations in the assumption.

Appendix

The relations between the degree of assets substitution \((s)\) with the life of the pegged exchange rate system \((t_N)\)

Lemma 1.

For any positive parameters of \(c, \ x>0\), Let \(g_s(t) \equiv -1 + (t.c.x + t.c+1) e^{-t(x+1)c}\).

Then for any \(t>0, g_s(t)<0\).

Proof.

Since \(g_s(0)=0\), \(dg_s(t)/dt = - t.c^2 (x+1)^2 e^{-t(x+1)c} < 0\), namely \(g_s(t)\) is a negative monotonic decreasing function with respect to \(t\). Therefore all \(t>0\), we have \(g_s(t)<0\).

QED.

Lemma 2.

For any positive parameters of \(c, \ x>0\), Let

\[ f_s(t) \equiv - t.c(x+1)(1 + e^{-t(x+1)c}) + 2(1 - e^{-t(x+1)c}) \]

Then for any \(t>0, f_s(t)<0\).

Proof.

Since \(f_s(0)=0\),

\[ df_s(t)/dt = - c(x+1)(1 + e^{-t(x+1)c}) + t.c^2(x+1)^2 e^{-t(x+1)c} + 2c(x+1) e^{-t(x+1)c} \]

\[ = c(x+1)[-1 + (t.c.x + t.c+1) e^{-t(x+1)c}] \]
= c.(x+1) g, t < 0 (by lemma 1).

Namely \( f_x(t) \) is a negative monotonic decreasing function with respect to \( t \).

Therefore all \( t>0 \), we have \( f_x(t)<0 \). \( \text{QED} \).

**Lemma 3.**

For any positive parameters of \( c, x>0 \),  Let

\[
\begin{align*}
    f_u(t) & = t c \left( e^{-t(x+1)c} - 1 \right) + (1-x) - t c x \left( 1 - e^{-t(x+1)c} \right) 
\end{align*}
\]

Then for any \( t>0 \), \( f_u(t)<0 \).

**Proof.**

Since \( f_u(0)=0 \),

\[
\begin{align*}
    \frac{df_u(t)}{dt} & = - c \left( x \right) \left( e^{-t(x+1)c} \right) - t \left( 1 - x \right) c \left( 1 - e^{-t(x+1)c} \right) < 0.
\end{align*}
\]

Namely \( f_u(t) \) is a negative monotonic decreasing function with respect to \( t \).

Therefore all \( t>0 \), we have \( f_u(t)<0 \). \( \text{QED} \).

**Theorem 1.**

For any \( t_N>0, v_k<0, CA<0 \), we have:

\[
\begin{align*}
    \frac{dt_N}{ds} & = \frac{\lambda c^2}{-\Delta(t_N) \alpha \beta^2} \left[ f_u(t_N)CA + f_v(t_N)v_k \right] > 0.
\end{align*}
\]

**Proof.**

From (14) and (14c), we have:

\[
\begin{align*}
    \frac{dt_N}{ds} & = t_N \frac{c \lambda}{\alpha} e^{-\beta_t t_N} \left( \frac{\beta_1}{\beta_2} \right) + \frac{c \lambda}{\alpha \beta^2} \left[ CA - CA \frac{s \lambda}{\alpha} + 2v_k \right] (1-e^{-\beta t_{t_N}}) - t_N \frac{c^2 \lambda}{\alpha \beta^2} (CA + v_k)
\end{align*}
\]

\[\quad - \Delta(t_N)\]
To simplify the notations, define \( x \equiv s \lambda / \alpha \). Then parameters of \( \beta_2 \) and \( \beta_3 \) can be rewritten as follows:

\[
\beta_2 = c.s.\lambda/\alpha + c = c(x + 1) > 0,
\]

\[
\beta_3 = c[s.\lambda.CA/\alpha - v_k] = c(x.CA - v_k).
\]

Hence

\[
\frac{dt_N}{ds} = \frac{\lambda c^2}{-\Delta(t_N)\alpha \beta_2^3} \left[ f_u(t_N)CA + f_v(t_N)v_k \right]
\]

where \( f_u(t) \) and \( f_v(t) \) are defined in Lemmas 2 and 3, respectively. By Lemmas 2 and 3, as well as \( t_N > 0 \), we have \( f_u(t_N) < 0 \) and \( f_v(t_N) < 0 \). Therefore we know that no matter it is the capital account outflow shock \( v_k < 0 \), or the current account deficit shock \( CA < 0 \), we always have

\[
\Delta(t_N) < 0 \text{ and } \frac{dt_N}{ds} > 0. \textbf{QED.}
\]

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