# MODELLING THE IMPACT OF CHINA'S WTO MEMBERSHIP ON

# **INVESTMENT AND GROWTH:**

# A NEW FLEXIBLE KEYNESIAN APPROACH

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## **ABSTRACT**

The November 2001 accession of China to the WTO promises increased investment (already the world's second in 1998 and third in 1999) particularly from the EU into the country. This investment is crucial to China's expanding trade with the rest of the world and will contribute significantly to its growth. The paper focuses on this nexus, presents a new and flexible approach to modelling the impact of China's WTO membership on its investment and growth within the general framework of multi-sectoral economies (Tran Van Hoa, 1992), and applies it to study this anticipated impact using recent World Bank data. Our approach dominates in efficiency the CGE and other neo-classical methods (such as used in GTAP models) in its data-consistent structure.

The paper then briefly describes the fundamentals of the new two-stage hierarchical information (2SHI) or empirical Bayes estimation and forecasting theory (Tran Van Hoa, 1985, 1986a, 1993b, Tran Van Hoa and Chaturvedi, 1988, 1990, 1997), summarises its superior MSE properties for forecasts and simulation, and reports substantive empirical findings on China's investment and growth given its trade enhancement positions. As an illustration of applications of our approach, impact on China's growth over a 7-year timeframe of a price reduction and increased government spending, assumed as a result of the country's WTO membership, is also investigated and briefly its policy implications discussed.

#### 1 Introduction

The standard theories of economics, international finance, transnational corporations, and within the accounting framework of the United Nations System of National Accounts (SNA) stipulate that investment plays a crucial role in influencing microeconomic decisions and macroeconomic activity, national output growth and economic development. Investment also helps to shape fiscal and monetary policy (Dornbusch and Fischer, 1990) and economic reforms in many developed, newly industrialized and especially developing countries (World Bank, 1991). Corporate and private strategies for business development and expansion in a home or host economy depend on this crucial role in a Wiener-Granger causal sense. As a result, a rigorous study and discussions of the movements or trends of these economic aggregates and their empirical relationships either in a historical context or in future predictions are amply justified.

With its formal accession to the WTO in November 2001, China expects, as a result in the medium and long term, enhanced trade and investment and economic growth in that sequential order. The purpose of our paper is therefore to contribute to this important nexus study by exploring empirically the interdependence of the various 'engines of growth' and by using a new modelling approach integrating the conventional dynamic multi-equation Keynesian theory and the SNA93 framework.

To achieve our purpose in a novel way, the paper departs from the applied econometric modelling approaches using conventional multiple regressions, Haavelmo's simultaneous equations, Zellner's SUR, or subjective or data-inconsistent CGE and GTAP, and makes use instead of a fairly simple and flexible economy-wide modelling approach based on the calculus of differential analysis in economics (Tran Van Hoa, 1992a, 1992d) to provide the fundamental equations in the reduced form for macroaggregates of interest. The success of this new approach is assessed via its modelling performance, analytic and empirical.

Finally, the paper is a contribution to applications of recent advances in the econometric theory of forecasting and impact studies to better formulate forward planning/impact policy and strategies either in finance, economics, or business. These advances consist of our new empirical Bayes or hierarchical information (Tran Van Hoa, Tran Van Hoa, 1985, 1986a, 1993b, Tran Van Hoa and Chaturvedi, 1988, 1990) theories that have average MSE or Wald risk properties superior to other conventional methods including the OLS, the maximum likelihood, or the explicit (Baranchik, 1973) positive Stein-like (Anderson, 1984) methodologies.

The implications from our paper are twofold. First, if the modelling success of our new modelling approach is relatively superlative -- in terms of its empirical fit and turning point predictions -- then its superiority in impact studies for say China's WTO membership is confirmed, historically or in an ex ante sense. Secondly, if, based on the same model and dataset, a substantial improvement in impact outcomes is achieved by our 2SHI methods in relation to other conventional procedures currently in use, then our findings will, in addition, point to a new direction of rigorous modelling and impact study methodology not only for China's WTO membership but also for finance, economics, and business analysts in their corporate and individual planning applications to investment and growth.

#### **2** Trends in China's Main Economic Activities

In the investment area, China has actively sought foreign direct investment (FDI) and technology to promote its modernization efforts and accelerate its export trade capabilities since its opening up in 1978.

The total amount of incoming FDI rose from almost zero in that year to a high of about \$110 billion in 1993 and \$320 billion in 1999. Thus, China has become the world's third (second until 1998) largest recipient of FDI, and the largest recipient among developing countries. The FDI volume has hovered around \$40-50 billion in each of the past several years. China expects significantly increased inflows over the next five years following its WTO accession in November 2001 that would bring about the removal of a number of restrictions on foreign investment (OECD, 2001)

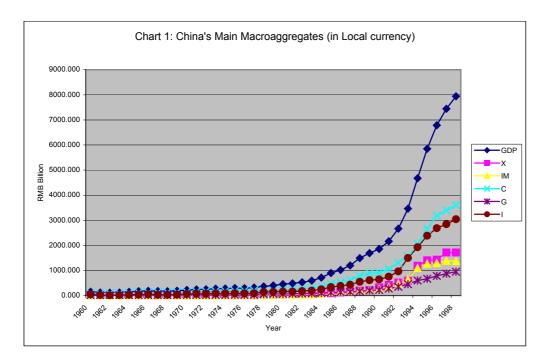
At present, China is on the threshold of crucial developments and policy decisions concerning FDI, and keen to increase the level, quality and source diversity of incoming FDI, particularly FDI from the OECD Member countries. The Chinese government also continues its efforts to adjust its policy and institutional framework, as well as investment promotion policies, to the requirements of the changing international and domestic circumstances (OECD, 2001).

Previous efforts by the OECD and China's Ministry of Foreign Economic Co-operation and Trade (MOFTEC) resulted in a conference in Xiamen, Fujian, in September 2000 to discuss the driving forces and economic effects of FDI on China's development, to exchange information and experiences on best FDI promotion practices, and to elaborate China's future FDI plans and policies. Our study below supplements these efforts from a methodological perspective and with important implications for more effective impact studies of China's WTO membership supported by historical data over three decades.

Chart 1 summarises the annual (1960-1998) movements of China's six main SNA macroaggregates: GDP, private consumption, gross fixed investment, government expenditure, exports, and imports (all at current prices). All these aggregates show an exponential growth path from 1960 to 1998 recording the well-known fast pace of China's recent development. The largest component of GDP is private consumption, followed by investment, exports and imports (generally with a trade surplus). Despite the role played by the government in the economy, its SNA share is the lowest as seen in Chart 1. Significantly, there is a slight sign of the Asia crisis' impact only in so far as exports and imports are concerned. Also, there seems to be a link between the trends of these macroaggregates.

While the impact of China's WTO membership can be studied via increased exports and imports alone, the apparent link between them and the rest of the economy's activities in an SNA93 context makes the task of study more complicated.

More specifically, to decompose this link into a more precise form for say a study of the impact of increased trade and investment from China's WTO membership or for other policy analysis, we have a number of options. These options can involve (a) a descriptive analysis of the graphs of these macroaggregates, their means, standard deviations and cross-correlations, or their shifts over time. We can also look at (b) the econometric interaction between trade and investment and other relevant causal economic activities in the economy. We adopt the second approach because of the inherent Marshallian nature of economic activities in which all are interrelated or interdependent.



Source: World Bank World Tables 2001.

# 3 Modelling China's Economy: A Multi-sectoral Flexible Keynesian Approach

Modelling an economy can take many approaches and forms with varying degrees of success from both a theoretical and empirical context. In an economy with interdependent sectors and activities, any of our six macroaggregates for China above could be argued to be dependent on many varied internal and external, economic and non-economic factors and in a linear, nonlinear, or mixed functional form. Consider for illustration in this paper a simple well-known generic five-equation Keynesian macroeconomic model of an open economy in the linear form as

$$C_{t} = \alpha 11 + \alpha 12Y_{t} + \alpha 13C_{t-1} + u1_{t}$$
 (1)

$$I_{t} = \alpha 21 + \alpha 22Y_{t} + \alpha 23Y_{t-1} + \alpha 24R_{t} + \alpha 25R_{t-1} + u2_{t}$$
(2)

$$X_t = \alpha 31 + \alpha 32Y_t + \alpha 33YW_t + \alpha 34PCN + \alpha 35PUS_t + u3_t$$
 (3)

$$IM_t = \alpha 41 + \alpha 42Y_t + \alpha 43YW_t + \alpha 44PCN + \alpha 45PUS_t + u4_t$$
 (4)

$$Y_t = C_t + I_t + G_t + X_t - IM_t$$
 (5)

where C = private final consumption expenditure, Y = gross domestic product or GDP, I = private gross fixed investment, G = government expenditure, X = exports of goods and services, IM = imports of goods and services, YW = US income (as a proxy for world income), PCN = general price deflator in China, PUS = general price deflator in the US (as a proxy for world prices), and R = US prime rate (as a proxy

for world interest rate). The  $\alpha$ 's denote the structural parameters, and the u's the error terms. All value variables are expressed in terms of their constant 1995 prices.

The model (1)-(5) is a dynamic macroeconomic model (Pindyck and Rubinfeld, 1991) for an open economy and takes into account (a) a partial adjustment process in consumption behaviour encompassing the hypotheses of relative and permanent income, liquid assets, wealth, and life cycles in the sense of Duesenberry, Friedman, and Modigliani, (b) a flexible accelerator investment behaviour, augmented by foreign capital borrowings (see for further detail Tran Van Hoa and Harvie, 1998) and user's costs, (c) trade openness through exports and imports regulated by foreign and domestic demand conditions and price relativities and (d) relevance of the government sector expenditure especially in transition economies.

In this model, the WTO membership impact is assumed to be transmitted through increased trade and investment, and this itself will be transmitted to increased growth and private consumption. In the model, consumption, investment, exports, imports and GDP are endogenous, and there are 9 exogenous and predetermined variables.

It can be verified that, using the order condition for identifiability or mathematical consistency in the theory of econometrics, all equations in the model are over-identified. As a result, the endogenous variable equations for say investment, consumption and growth can be written, instead of their linear form given traditionally in (1)-(5), in their complete differential form (see Allen, 1960) in the reduced form as (see Tran Van Hoa, 1992a and 1992d, Harvie and Tran Van Hoa, 1993)

$$\begin{split} \text{I}\%_t &= \text{a}11 + \text{a}12 \text{ C}\%_{t-1} + \text{a}13\text{Y}\%_{t-1} + \text{a}14\text{R}\%_t + \text{a}15\text{R}\%_{t-1} \\ &\quad + \text{a}16\text{Y}\text{W}\%_t + \text{a}17\text{PCN}\% + \text{a}18\text{PUS}\% + \text{a}19\text{G}\%_t + \text{e}1_t \\ \text{C}\%_t &= \text{a}21 + \text{a}22 \text{ C}\%_{t-1} + \text{a}23\text{Y}\%_{t-1} + \text{a}24\text{R}\%_t + \text{a}25\text{R}\%_{t-1} \\ &\quad + \text{a}26\text{Y}\text{W}\%_t + \text{a}27\text{PCN}\% + \text{a}28\text{PUS}\% + \text{a}29\text{G}\%_t + \text{e}2_t \\ \text{Y}\%_t &= \text{a}31 + \text{a}32 \text{ C}\%_{t-1} + \text{a}33\text{Y}\%_{t-1} + \text{a}34\text{R}\%_t + \text{a}35\text{R}\%_{t-1} \\ &\quad + \text{a}36\text{Y}\text{W}\%_t + \text{a}37\text{PCN}\% + \text{a}38\text{PUS}\% + \text{a}39\text{G}\%_t + \text{e}3_t \end{aligned} \tag{6c}$$

where I%, C%, Y%, R%, YW%, PCN%, PUS%, and G% indicate the rate of change of I, C, Y, R, YW, PCN, PUS and G respectively, a's indicate the reduced form parameters, and e's are the new error terms.

The 3 equations in (6) characterize the investment, consumption and growth relationships from the illustrative five-equation macroeconomic model given in Eqts (1)-(5). In contrast to the conventional unidirectional analysis, the impact of enhanced trade due to China's WTO membership on investment, consumption and growth can be only seen indirectly through the exogenous and predetermined variables. By conventional definition, the impact parameters from these equations are in fact either static (or dynamic) elasticities associated with either current (or lagged) variables included in it.

The derivation of (6) by means of total differentiation of an arbitrarily functional relationship is simple and, more importantly, consistent with the procedure usually adopted for the neoclassical macroeconomic models of the applied or CGE kind. In these neo-classical models, the endogenous and exogenous variables in the economy are linked by a (usually first order) approximate transmission mechanism in terms of the elasticities. There are however at least five important differences between our equations given in (6) above and the specifications from applied or CGE Johansen-class models

First, in our case, the important linking elasticities have to be estimated for the model as a whole using economic time-series data and possibly other extraneous (prior) information such as policy switches or external non-economic factors. Our equations given in (6) thus are completely data-based, although clearly we do not preclude the use of prior or extraneous information (in the form of an oil crisis or a major war for example) in the equations in other theoretical or judgemental contexts.

Secondly, in view of the above arguments, our model is capable of accommodating sub- and add-factors as well as structural change and other institutional considerations (for a discussion supporting the use of these factors in macroeconomic models, see Johansen, 1982).

Thirdly, our equations must be mathematically consistent as required by the identifiability conditions for complete systems of structural simultaneous equations in the theory of econometrics.

Fourthly, by its construct, our modelling approach encompasses a wide class of linear and nonlinear multi-equation econometric models in which the exact functional form of each of the individual structural equations is as usual unknown or needs not be specified.

Finally, for an important group of economic variables whose first differences in logs are approximately equivalent to the rates of change, our equations by their construct include as the special cases the Granger-Wiener short term causality if these rates of changes are I(0) and the co-integration or long-term equations of the Engle-Granger (1987) class (see Tran Van Hoa, 1993c, and Harvie and Tran Van Hoa, 1993, for further detail) if the rates of change are I(1).

To evaluate the performance of the equations of interest in (6) in our macroeconomic model (1)-(5) and our new impact study methodology using real-life data from China in recent years, we have fitted the equations (6) to data for the period 1966 to 1998. This will optimally produce the necessary elasticity estimates. These estimates can then be used in a comparative study which is based on stochastic simulation to measure the relative MSE performance or operational accuracy of our modelling equations. These equations can finally be used for a study of the impact of China's WTO membership under different and plausible scenarios of shocks or policy regimes.

# 4 Impact and Forecasting Study: Alternative Estimating Methodologies

The equations in differential and reduced form as given in (6) can be written more generally with a sampling size T and k independent variables (possible causes) in matrix notation as

$$y = Z \quad \beta + u$$

$$(Tx1) \quad (Txk) \quad (kx1) \quad (Tx1)$$

$$(7)$$

where y=I% or Y%, Z=the rate of changes of the exogenous and predetermined variables (both static and dynamic),  $\beta$  =the parameters, and u the disturbance satisfying all standard statistical assumptions.

To estimate (7) which is essentially a general linear model (7) for structural or behavioral analysis or for direct forecasting and policy analysis (see Pindyck nd Rubinfeld, 1991), we can use the OLS, or, at a more efficient level, any of the explicit (Baranchik, 1973) Stein or Stein-rule methods as described below.

More specifically, using (7), the basic and most well known method to produce estimates and forecasts of y (or I%) is the OLS estimator of  $\beta$  (denoted by b) and is written as

$$b = (Z'Z)^{-1}Z'y \tag{8}$$

A more sophisticated and efficient method is the explicit Stein estimator of ß (Baranchik, 1973) that is given by

$$\beta s = [1 - c(y-Zb)'(y-Zb)/b'Z'Zb] b$$

$$= [1 - c(1-R^2)/R^2] b$$
(9)

where c is a characterizing scalar and defined in the range 0 < c < 2(k-2)/(T-k+2), and R<sup>2</sup> is the square of the sample multiple correlation coefficient.

A still more efficient method is the explicit positive-part Stein estimator of  $\beta$  (Anderson, 1984) which is defined as

$$\beta + s = [1 - \min\{1, c(y-Zb)'(y-Zb)/b'Z'Zb\}] b$$

$$= [1 - \min\{1, c(1-R^2)/R^2\}] b$$
(10)

A new method to obtain estimates and forecasts of ß in (7) with better properties has been proposed (see Tran Van Hoa, 1985, Tran Van Hoa and Chaturvedi, 1988 and 1990). It is in a class of explicit improved Stein-rule or empirical Bayes [also known as two-stage hierarchical-information (2SHI)] estimators for some linear regression models. This estimator includes the explicit Stein and the double k-class (Ullah and Ullah, 1981) estimators as subsets (Tran Van Hoa, 1993b). Other applications of the Stein, Stein rule, and 2SHI estimators to linear regression models with non-spherical disturbances and to Zellner's seemingly unrelated regression model have also been made (see Tran Van Hoa et al, 1993a, in the case of regressions with nonspherical disturbances, and Tran Van Hoa, 1992b, and 1992d, in the case of seemingly unrelated regressions).

The explicit 2SHI estimator is defined as

$$\beta h = [1 - c(1-R^2)/R^2\} - c(1-R^2)/\{R^2(1+c(1-R^2)/R^2)\}] b$$
(11)

and its positive-part counterpart (Tran Van Hoa, 1986a) is given by

$$\beta + h = [1 - \min\{1, c(1-R^2)/R^2\} - \{1/((R^2/c(1-R^2)) + 1)\}] b$$
 (12)

While all the estimators given above can be applied to the general linear model (7) for structural and forecasting analysis, their relative performance in terms of historical, ex post or ex ante (Pindyck and

Rubinfeld, 1991) forecasting MSE can differ. Thus, it is well-known that, in MSE and for  $k \ge 3$  and  $T \ge k + 2$ ,  $\beta$ s dominates (that is it performs better in forecasting MSE) b, and  $\beta$ s is dominated by  $\beta$ +s (Baranchik, 1973, Anderson, 1984). However, it has also been demonstrated (Tran Van Hoa, 1985, Tran Van Hoa and Chaturvedi, 1988) that, in MSE,  $\beta$ s dominates both b and  $\beta$ s, and more importantly,  $\beta$ +h dominates  $\beta$ +s (Tran Van Hoa, 1986a).

A further important path-breaking result of the 2SHI theory has recently been proved (see Tran Van Hoa and Chaturvedi, 1997): the dominance of the 2SHI over the OLS and Stein exists anywhere in the range 0 < c < 2(k-1)/(T-k). This indicates that the 2SHI method produces better (in terms of smaller Walk risk or generalized Pitman nearness) estimates and forecasts even if the estimating and forecasting equation has only one independent variable in it. The condition for the optimal Stein dominance in the linear equation up to now requires that 0 < c < 2(k-2)/(T-k+2) [see Anderson, 1984].

While some application of these forecasting methodologies to predictions of economic activities in some developed countries such as Australia (see Tran Van Hoa, 1992d) has been made, the extent of the significance of the MSE dominance, or equivalently, the informational gain or relative forecasting success between the alternative estimators above has not been investigated explicitly within an open trade theoretical framework and an empirical context using more recent economic data for the major economies in East Asia. This issue is taken up in the study below for one of the fastest growth economies in the world in recent years but with highly fluctuating investment and being very sensitive to foreign trade and capital flows in the region (see Tran Van Hoa and Harvie, 1998).

Another interesting feature of our study is that, since all data are annual and have as usual a small sample size, our study is therefore designed to look at the finite sample performance of alternative impact study methods.

Finally, since the poor quality of economic data from the Asian countries and other less developed countries (LDC) economies is well known, one by product of our study is that we in fact investigate the performance of the alternative forecasts in the case of serious measurement errors on the variables of the macromodel of an economy however it is defined.

The substantive findings reported below are based on the five-equation macroeconomic model described earlier in (1)-(5), and the appropriate estimating equations (6) to produce elasticity parameters or the equations to study the impact of China's WTO membership on investment, consumption and growth as given in (6). In addition, a number of well-known methods is used to compare their relative performance for decision analysis.

# 5 Real-Life Performance of Alternative Methodologies

In our study, we have fitted the three equations (investment, consumption and growth) in differential and reduced form (6a-6c) of the model (1)-(5) to annual data from China. The original dataset is from 1961 to 1998, but the effective (ie, after allowing for missing or statistically incompatible data) sample period is 1962 to 1998, giving, when the dynamic (lag) structure is taken into account, a sample size of up to 37 observations for each variable. In our comparative study, only the OLS or ML, the positive-part Stein, and the positive-part 2SHI forecasts of investment are used to produce elasticities for impact study.

The data for our study are in real terms at the constant 1995 prices and obtained from the 2000 World Bank World Tables Industrial Countries and East Asia databases, using Australia's data express (DX)

extracting procedure. The performance of our reduced form equations is determined solely from their good fit, correct turning point predictions and improved forecasting MSE.

The possible effects of structural instability (economic dynamism) and uncertainty (due to financial or economic crises) on China's investment, consumption and growth have also been taken into account. This is achieved by modelling these macroaggregates in the short (one year), medium (two years) and long-terms (3 or more years), and also subjecting our impact to wide fluctuations over the study period.

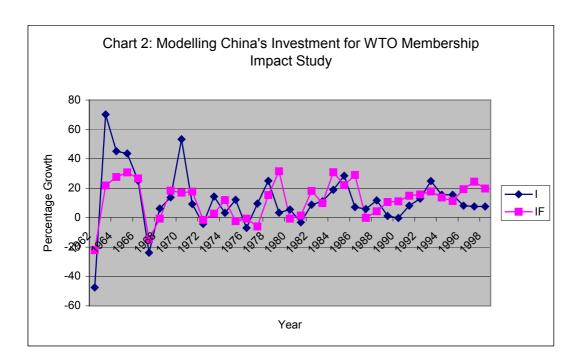
The simulation results from this study (not shown here) support the analytical results discussed in Section 3 in all scenarios (see also other evidence in Appendix). More specifically, our methodologies indicate that they are specially suited to study the impact of the factors affecting China's trade, investment, consumption and growth especially in the long term.

# 6 Measuring the Impact of China's WTO Membership on Investment, Consumption and Growth

## **Possible Impact on China's Investment**

In order to study the impact of China's WTO membership on the country's investment, it is necessary to study the determinants (whether WTO-induced or not) in our specified model that have directly or indirectly affected this investment in an historical context. The relevant determinants of investment based on our simple model (1)-(5) are given in Eqt (6a). The impact of the WTO membership on China's investment is transmitted directly via enhanced trade (exports and imports) but indirectly through these determinants at the end of the transmission mechanism. A good equation in this case would then be able to provide better outcomes for future impact or ex ante studies on investment.

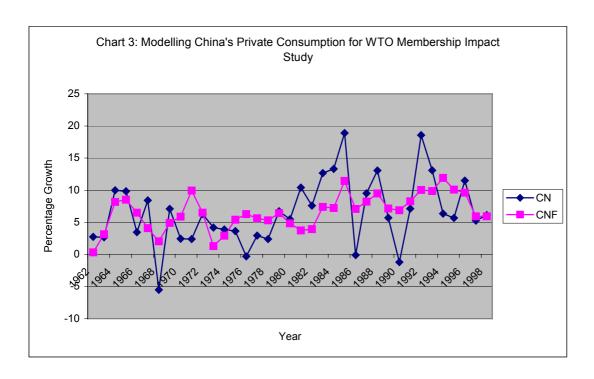
The historical forecast movements of China's investment are econometrically efficient and emulate well its actual fluctuations (peaks and troughs) during the period under study, 1962-98 (see Chart 2). While there are some underestimation of the actual peaked investment in the early 1960s, there is some indication of more minor overestimation in the recent years due possibly to other relevant but omitted (internal or external) factors from our model. Of special interest to us is however the ability of our estimated model to mimic not only the trends but also the turning points of the observed investment data over nearly 4 decades, even though we conceded earlier that our model is simply an illustration of the performance of our new modelling methodologies for impact study.



## **Possible Impact on China's Private Consumption**

Modelling China's private consumption (6b) for impact study and based on our illustrative Keynesian model (1)-(5) has not been as successful in accurately predicting the peaks and troughs as in the case of investment (see Chart 3). This may be understandable in the sense that this underestimation of consumption occurred during the period of great constitutional changes. These changes include the opening up of China to the world in 1972, the internal turmoil of 1989, and the Gulf War of the early 1990s. In addition, expected conspicuous consumption in China has been fairly high in the decades of high growth (1980s and early 1990s) and political stability in the country.

The last explanation needs further clarification. In fact, it also is our observation that in economies of fairly sustained high growth rates, sometimes over-hyped expectations of better things seem to override unrealistically other considerations (see also Duncan Hewitt (2001), *Chinese Hope for Consumer Revolution*, http://news.bbc.co.uk.hi)/english/world/asia-pacific). It is within this context of consumer's behaviour that our model which is based on the Keynesian-Friedman-Modigliani theory should be amended to take this observation into account in future modelling and impact studies.



# **Possible Impact on Growth**

In contrast to the performance of the private consumption equation (6b) discussed above, our estimated equation (6c) for China's growth over the period of more than three decades (1962-98) appears to fare very well (see Chart 4). These findings also indicate that the equation is particularly suitable for WTO membership impact study. It should be noted that the sampling period is a period of great upheavals in the country that also was subject to big changes or shocks outside it. Some of these shocks are well-known such as the opening up of China to the outside world, the two great oil crises of 1974 and 1981, the crash of the stock market in the US in 1987, the Gulf War in 1991, the internal turmoil of 1989, and the Asia crisis of 1997. While these shocks and the consumer's hype may have affected our estimated private consumption, they are capably and fully accommodated in our estimated growth equation.

Thus, in spite of these so-called outliers, our estimated growth equation could mimic almost all fluctuations (including the peaks and troughs) of China's spectacular and widely fluctuating output growth during 1962-98. On the basis of these findings, our model as given in (1)-(5) should be capable of calculating fairly accurately the impact of China's WTO membership on the country's principal economic objective: its increasing standard of living. The impact is transmitted significantly through the variables specified for the model and given in the reduced form equation (6c).

There is however no reason why other relevant variables or the sub- and add-factors in the sense of Jorgensen cannot be integrated into the model (1)-(5) for a better study of the impact of the WTO membership on China's growth or standard of living in the future.



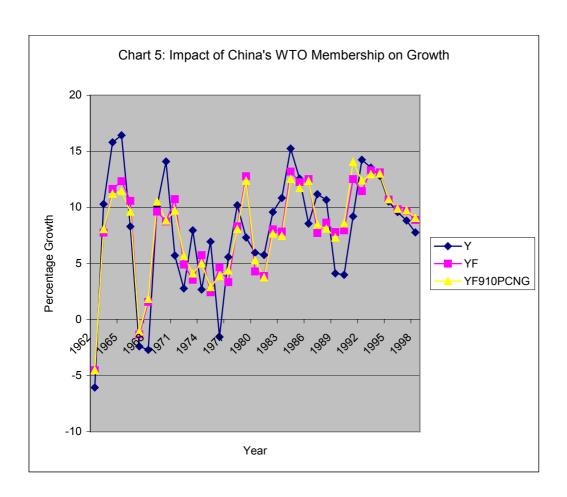
# Case study: WTO Membership Impact (via Price Reduction and Government Spending) on China's Growth

Below we have used our estimated reduced form equation (6c) for China's growth to study the perceived impact of the country's 2001 WTO membership on its standard of living or growth rates over a number of years. The analysis is historical but its translation to impact studies for the post-WTO accession period is straightforward. The methodological justification for this argument is that good historical or ex post outcomes will produce good ex ante results (Pyndyck and Rubinfeld, 1991).

As a result of the WTO membership, it is assumed that opening up the Chinese market to the outside world and accompanying reform (and change of culture) will reduce the general price level (the CPI) in China by 10 per cent. This will boost domestic spending due to reduced commodity prices and increase government revenue and subsequent increased government spending. Assume that the spending boost is also 10 per cent. Assume also that the impact started in 1991 for historical studies (or equivalently in 2002 for ex ante studies). The estimated impact of the WTO membership on China's growth rate under this scenario is depicted in Chart 5.

From Chart 5 we note that the a 10 per cent reduction in the CPI and a 10 per cent increase in government expenditure would have a mixed effect initially on China's growth rate. The effect becomes clearer and more uniform after a few years.

For example, in the first two years after the changes (that is, the WTO membership), there would be a slow-down (that is, lower but still very much positive rates) in China's growth. This may be due to restructuring in policy and community perception in the country. But six years after the changes, China's growth will attain an increase of 10.1 per cent higher than that in the no-change (no WTO membership) state. And seven years after the changes, China's growth will be 16.5 per cent higher than that in the no-change state.



## 7 References

Allen, R.G.D. (1960), Mathematical Analysis for Economists, London: Macmillan.

Anderson, T.W. (1984), An Introduction to Multivariate Statistical Analysis, 2nd Edition, New York: Wiley.

Baranchik, A.J. (1973), "Inadmissibility of Maximum Likelihood Estimators in some Multiple Regression Problems with Three or More Independent Variables", Annals of Statistics, Vol. 1, 312-321.

Chaturvedi, A. and Tran Van Hoa (1997), "The dominance of the 2SHI estimator under generalized Pitman nearness", Communications in Statistics, Vol. 26, Issue 5, 1997. pp. 1227-1238.

Dornbusch, D. and Fischer, S. (1990), Macroeconomics, 5th Edition, Sydney: McGraw Hill.

Engle, R.F. and Granger, C.W.J. (1987), "Co-integration and Error Correction: Representation, Estimation and testing", Econometrica, Vol. 55, 251-276.

Harvie, C. and Tran Van Hoa (1993), "Long Term Relationships of Major Macrovariables in a Resource-Related Economic Model of Australia: a Cointegration Analysis", Energy Economics, Vol. 15, 257-262.

Johansen, L. (1982), "Econometric Models and Economic Planning and Policy: Some Trends and Problems", in M. Hazewinkle and A.H.G. Rinnooy Kan (eds.), Current Developments in the Interface: Economics, Econometrics, Mathematics, Boston: Reidel.

OECD (2001), Foreign Direct Investment in China, www/oecd/org/oecd/pages/.

Pindyck, R.S. and Rubinfeld, D.L. (1991), Econometric Models and Economic Forecasts, Sydney: McGraw-Hill.

Tran Van Hoa (1985), "The Inadmissibility of the Stein Estimator in Normal Multiple Regression Equations", Economics Letters, Vol. 19, 39-42.

Tran Van Hoa (1986a), "The Inadmissibility of the Stein Estimator in Normal Multiple Regression Models: Analytical and Simulation Results", 15th Anniversary of the NBER-NSF Seminar on Bayesian Inference in Econometrics, ITAM, Mexico City, 16-18 January 1986.

Tran Van Hoa (1986b) "Effects of Monetary and Fiscal Policy on Inflation: Some Evidence from the J-Test", Economics Letters, Vol. 22, 187-190.

Tran Van Hoa and Chaturvedi, A. (1988), "The Necessary and Sufficient Conditions for the Uniform Dominance of the Two-Stage Stein Estimators", Economics Letters, Vol. 28, 351-355.

Tran Van Hoa and Chaturvedi, A. (1990), "Further Results on the Two-Stage Hierarchical Information (2SHI) Estimators in the Linear Regression Models", Communications in Statistics (Theory and Methods), Vol. A19, No. 12, 4697-4704.

Tran Van Hoa (1992a), "Modelling Output Growth: A New Approach", Economics Letters, Vol. 38, pp. 279-284.

Tran Van Hoa (1992b) "Energy Consumption in Thailand: Estimated Structure and Improved Forecasts to 2000" (in Thai), Thammasat Economic Journal (Thailand), Vol. 10, 55-63...

Tran Van Hoa (1992c), "A Multi-equation Model of Energy Consumption in Thailand", International Journal of Energy Research, Vol. 16, pp. 381-385.

Tran Van Hoa (1992d), "A New and General Approach to Modelling Short-Term Interest Rates: With Application to Australian Data 1962-1990", Journal of Economics and Finance: Proceedings, Vol. 16. 327-335.

Tran Van Hoa (with Chaturvedi, A. and Shukla, G.) (1993a), "Performance of the Stein-rule Estimators when the Disturbances are Misspecified as Spherical", Economic Studies Quarterly (Japan), Vol. 44, 601-611.

Tran Van Hoa (1993b), "The Mixture Properties of the 2SHI Estimators in Linear Regression Models", Statistics and Probability Letters, Vol. 16, 111-115.

Tran Van Hoa (1993c), "Effects of Oil on Output Growth and Inflation in Developing Countries: The Case of Thailand 1966:1 to 1991:1", International Journal of Energy Research, Vol. 17, 29-33.

Tran Van Hoa and Chaturvedi, A. (1997), "Performance of the 2SHI Estimator under the Generalized Pitman Nearness Criterion", Communications in Statistics (Theory and Method), Vol. 26, Issue 5, 1227-1238.

Tran Van Hoa and Harvie, C (1998), Causes and Impact of the Asian Financial Crisis, London: Macmillan.

Ullah and Ullah (1978), "Double k-class Estimators of Coefficients in Linear Regression", Econometrica, Vol. 46, pp. 705-722.

World Bank (1991), The Challenge of Development, World Development Report, Oxford: Oxford University Press.

World Bank World Tables (2000).

# PERFORMANCE OF THE 2SHI ESTIMATOR UNDER THE GENERALISED PITMAN NEARNESS CRITERION

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#### **ABSTRACT**

The paper considers an extension of Tran Van Hoa's family of 2SHI (two stage hierarchical information) estimators for the coefficient vector of a linear regression model and derives the conditions for the dominance of the 2SHI estimator over the OLS and Stein rule estimators under a Generalized Pitman Nearness (GPN) criterion when the disturbance variable is small.

#### 1. INTRODUCTION

In 1985, Tran Van Hoa proposed a family of 2SHI (two stage hierarchical information) estimators for the coefficient vector of the linear regression model. These 2SHI estimators were demonstrated to dominate in average mean squared errors (MSE) the OLS and the Stein estimators. A number of applications of the 2SHI estimators in empirical economic studies based on static and dynamic regression models where the 2SHI dominance was calculated have also been reported (see Tran Van Hoa 1992a, 1992b, 1993).

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In 1990 and 1993, Tran Van Hoa and Chaturvedi extended the 2SHI further and considered a more general family of 2SHI estimators. They obtained the conditions for the dominance of the 2SHI estimator over the OLS and Stein rule estimators under a quadratic loss function. In those studies, the criterion of relative MSE or risks in the sense of Wald was adopted.

The MSE criterion is only one of many criteria that can be used in the studies of this kind. The Pitman nearness criterion is another concept that has been developed and used by several researchers for a comparison of alternative estimators, see Keating and Mason (1985), Rao et. al. (1986), Khatree (1987) and Peddada (1987), to cite a few. A special feature of this criterion is that it does not require the existence of the moments of the estimator and is less sensitive to the tail behaviour of the sampling distributions of the estimator. Rao et. al. (1986) and Keating & Mason (1988) considered a Generalized Pitman Nearness (GPN) criterion and analysed the performance of the Stein rule estimator in comparison to the Maximum Likelihood Estimator (MLE) for the mean of the multivariate normal distribution using extensive numerical studies. Sen et. al. (1989) derived the dominance condition for the Stein rule estimator over the MLE under a GPN criterion (see also Keating & Czitrom (1988) and Mason et. al. (1990)).

The main objective of the present paper is to establish the dominance of the 2SHI estimator over the OLS estimator and the Stein rule estimator under a GPN criterion. Since the technique adopted in Sen et. al. (1989) is quite involved and leads to fairly complicated expressions, we have adopted a simple methodology based on the small disturbances approximations.

#### 2. THE MODEL AND THE ESTIMATORS

Consider the linear regression model

$$y = X\beta + \sigma u \tag{1}$$

where y is a Tx1 vector of observations on the dependent variable, X is a Txk matrix of observations on k independent variables with full column rank,  $\beta$  is a kx1 vector of unknown regression coefficients and u is a Tx1 random vector following a multivariate normal distribution N(0,IT) independent of X and  $\sigma^2(>0)$  is the disturbance variance.

In 1990 and 1993, Tran Van Hoa and Chaturvedi generalized the work by Tran Van Hoa (1985) and proposed the following family of explicit 2SHI estimators  $\hat{\beta}$  h for the coefficient vector  $\beta$ :

$$\hat{\beta}_{h} = \left[ 1 - cw \frac{(1-R^2)}{R^2} - c(1-w) \frac{R^2}{R^2 \{1+c^* (1-R^2)/R^2\}} \right] b, \tag{2}$$

where  $b = (X'X)^{-1}X'y$  is the OLS estimator of  $\beta$ ,  $R^2 = (b'X'Xb/y'y)$  is the coefficient of determination corresponding to a no intercept model and w ( $0 \le w \le 1$ ),  $c(\ge 0)$  and  $c^*$  ( $\ge 0$ ) are the characterizing scalars.

We can equivalently write the estimators  $\hat{\beta}_h$  as:

$$\hat{\beta}_{h} = \left[ 1 - cw \frac{(y - Xb)'(y - Xb)}{b'X'Xb} - c(1 - w) \frac{(y - Xb)'(y - Xb)}{b'X'Xb + c^{*}(y - Xb)'(y - Xb)} \right] b,$$
(3)

It can be verified that, when c\*=0 or w=1, the 2SHI estimator  $\hat{\beta}_h$  reduces to the following Stein rule estimator  $\hat{\beta}_S$ .

$$\hat{\beta}_{S} = \left[1 - \frac{(y - Xb)'(y - Xb)}{b'X'Xb}\right] b. \tag{4}$$

#### 3. COMPARISON OF THE ESTIMATORS

For the comparison of the estimators, let us consider the quadratic loss function

$$M(\stackrel{\wedge}{\beta}) = (\stackrel{\wedge}{\beta} - \beta)' X' X (\stackrel{\wedge}{\beta} - \beta).$$

Then, following Rao et. al. (1986), the formal definition of the GPN criterion is given as follows:

#### **DEFINITION:**

For any two estimators  $\hat{\beta}$  and  $\widetilde{\beta}$  of  $\beta$ , under the loss function above, the estimator  $\hat{\beta}$  is said to be Pitman closer to the estimator  $\beta$  if

$$P[M(\widetilde{\beta}) - M(\widehat{\beta}) \ge 0] \ge \frac{1}{2}$$
.

#### THEOREM:

Under the assumption of small disturbances variance, up to order  $0(\sigma)$ , we have

$$P[M(b) - M(\hat{\beta}_h) > 0] = \frac{1}{2} + \frac{\sigma}{(2\pi\tau)^{1/2}} [(k-1) - \frac{c}{2} (T-k)].$$
 (5)

$$P[M(\hat{\beta}_{S}) - M(\hat{\beta}_{h}) > 0] = \frac{1}{2} - \frac{\sigma}{(2\pi\tau)^{1/2}} [(k-1) - c(T-k)].$$
 (6)

See Proof in the Appendix.

The above theorem implies that, up to the order of our approximations, under the GPN criterion, the 2SHI estimator  $\hat{\beta}$  h dominates the OLS estimator b whenever

$$0 < c < \frac{2(k-1)}{(T-k)}, k > 1;$$
 (7)

whereas,  $\hat{\beta}$  h dominates the Stein rule estimator  $\hat{\beta}$  s whenever

$$c > \frac{(k-1)}{(T-k)}, k > 1;$$
 (8)

The dominance conditions (7) and (8) show that under the GPN criterion the 2SHI estimator dominates both the OLS estimator as well as the Stein rule estimator whenever c lies in interval [(k-1)/(T-k),2(k-1)/(T-k)] and k>1. Thus, if k>1, it is possible to improve upon the Stein rule estimator by using the 2SHI estimator.

#### 4. SIMULATION RESULTS ON THE PERFORMANCE OF THE 2SHI

The performance, by means of a simple simulation study, of the 2SHI estimator over the OLS and Stein estimators in some linear regression models is given in the table below. The 27 different models are the reduced form equations of growth in Australia derived from a standard 4-equation Keynesian macroeconomic model for an open economyused. These models are characterized by different representative values of (a) the weight w, (b) the sample size T, and (c) the disturbance variance  $\sigma^2$ . In these models, w=(0.2, 0.5, and 0.8), T=(10,14 and 16), k=8, and c=c\* = 1.5(k-1)/(T-k). The raw data for X are actual annual economic data on GDP, consumption, investment, government expenditure, exports and imports and obtained from the international 1995 DX database. The magnitude of  $\sigma^2$  are defined over the range (0.01, 1, and 100). The simulation results are based on the averages from 100 statistical trials (larger numbers of trials have been tried but the findings remain basically unchanged). The dominance

between say b and  $\hat{\beta}_s$ , denoted by  $R(b/\hat{\beta}_s)$ , is computed as  $100[M(b)-M(\hat{\beta}_s)]/M(\hat{\beta}_s)$  where M(b) and  $M(\hat{\beta}_s)$  are the average loss of b and  $\hat{\beta}_s$  respectively. The calculation of the relative risk is similar for  $R(b/\hat{\beta}_h)$  and  $R(\hat{\beta}_s/\hat{\beta}_h)$ .

From these simulation results, the 2SHI estimator dominates both the OLS and Stein estimators in all models. The smallness of  $\sigma$  as discussed above in the paper can have a fairly wide range of values, from 0.1 to 10 in our simulation study. The dominance does not seem to be greatly affected by these different values of  $\sigma$ .

TABLE I Performance of the 2SHI Estimators over the OLS and Stein: Simulation Results

Average R <sup>2</sup>	erage R <sup>2</sup> (for all models) =								
	0.980	0.972	0.962	0.962	0.816	0.807	0.941	0.722	0.733
Weight w = T =	0.20	10			14			16	
$\sigma^2 =$	0.01		100	0.01		100	0.01		100
σ =	0.01	1	100	0.01	1	100	0.01	1	100
$R(b/\stackrel{\wedge}{\beta}_S)$	4.51	4.85	6.97	3.85	18.71	19.80	4.80	20.42	19.02
$R(b/\stackrel{\wedge}{\beta}_h)$	22.18	23.48	25.01	20.80	95.67	115.74	26.41	122.95	119.49
$R(\stackrel{\wedge}{\beta}_{S}/\stackrel{\wedge}{\beta}_{h})$	16.90	17.78	16.86	16.32	64.83	80.08	20.61	85.15	84.42
Weight w =	0.50								
T =		10			14			16	
$\sigma^2 =$	0.01	1	100	0.01	1	100	0.01	1	100
$R(b/\beta_s)$	11.39	11.95	12.87	9.97	48.07	53.14	12.53	56.24	53.97
$R(b/\stackrel{\wedge}{\beta}_h)$	21.91	22.92	24.74	20.64	92.94	112.42	26.16	119.14	116.84
$R(\stackrel{\wedge}{\beta}_s/\stackrel{\wedge}{\beta}_h)$	9.45	9.80	10.52	9.70	30.30	38.70	12.11	40.26	40.84
Weight w =	0.80								
T =		10			14			16	
$\sigma^2 =$	0.01	1	100	0.01	1	100	0.01	1	100
$R(b/\stackrel{\wedge}{\beta}_S)$	18.24	19.22	20.73	16.51	70.10	84.81	20.93	84.72	91.80
$R(b/\stackrel{\wedge}{\beta}_h)$	22.25	23.54	27.98	20.82	97.85	116.87	26.44	119.83	121.25
$R(\hat{\beta}_{s}/\hat{\beta}_{h})$	3.39	3.62	6.00	3.70	16.32	17.35	4.56	19.01	15.36

#### **BIBLIOGRAPHY**

Keating, J P and V Czitrom (1989), A Comparison of James Stein Regression with Least Squares in the Pitman Nearness Sense, *Journal of Statistical Computation and Simulation*, 34, pp. 1-9.

- Keating, J P and R L Mason (1985), Pitman's Measure of Closeness, Sankhya, Series B, 47, pp. 22-32.
- Keating, J P and R L Mason (1988), James Stein estimation from an Alternative Perspective, *American Statistics*, 42, pp. 160-164.
- Khattree, R (1987), On Comparison of Estimates of Dispersion Using Generalized Pitman Nearness Criterion, *Communications in Statistics (Theory and Methods)*, 16, pp. 263-274.
- Khattree, R and S D Peddada (1987), A Short Note on Pitman Nearness for Elliptically Symmetric Estimators, *Journal of Statistical Planning and Inference*, 16, pp. 257-260.
- Mason, R L, Keating, J P, Sen, P K and N W Blaylock (1990), Comparison of Linear Estimators using Pitman's Measure of Closeness, *Journal of American Statistics Association*, 85, pp. 579-581.
- Rao, C R, Keating, J P and R L Mason (1986), The Pitman Nearness Criterion and its Determination, *Communications in Statistics (Theory and Methods)*, 15, pp. 3173-3191.
- Sen, P K, Kubokawa, T and A K Md.E. Saleh (1989), The Stein Paradox in the Sense of Pitman Closeness, *Annals of Statistics*, 17, pp. 1375-1386.
- Tran Van Hoa (1985), The Inadmissibility of the Stein Estimator in Normal Multiple Regression Equations, *Economic Letters*, 19, pp. 39-42.
- Tran Van Hoa (1992a), Energy Consumption in Thailand: Estimated Structure and Improved Forecasts to 2000, (in Thai), *Thammasat Economic Journal*, Vol 10, pp. 55-63.
- Tran Van Hoa (1992b), A New and General Approach to Modelling Short Term Interest Rates: With Applications to Australian Data 1962-1990, Proceedings of the *Journal of Economics and Finance*, Vol 16, pp. 327-335.
- Tran Van Hoa (1993), Estimation and Inference in Dynamic Econometric Equations, Proceedings of the *American Statistical Association*, Section on Bayesian Statistical Science, pp. 241-248.
- Tran Van Hoa and A Chaturvedi (1990), Further Results on the Two Stage Hierarchical Information Estimators in the Linear Regression Models, *Communications in Statistics (Theory and Methods)*, 19, pp. 4697-4704.
- Tran Van Hoa and A Chaturvedi (1993), Asymptotic Approximations to the Gain of the 2SHI over Stein Estimators in Linear Regression Models when the Disturbances are Small, *Communications in Statistics (Theory and Methods)*, Vol 22, pp. 2777-2782.

#### PROOF OF THEOREM

Let us define

$$z = (X'X)^{-1/2} X'u, \quad \delta = (X'X)^{-1/2} \beta, \quad ,\tau = \delta'\delta,$$

$$v = u' [I_T - X(X'X)^{-1} X'] u.$$

Then  $z\sim N(0,I_k)$  and  $v\sim x^2$  distribution with (T-k) degrees of freedom independently of z. Then, to order  $0(\sigma^4)$ , we have

$$(b-\beta)'X'X(b-\beta)-(\stackrel{\wedge}{\beta}_h-\beta)'X'X(\stackrel{\wedge}{\beta}_h-\beta)$$

$$= 2c\sigma^3 \frac{v}{\tau} \left[ \delta' z + \sigma \left\{ z' z - \frac{2}{\tau} \left( \delta' z \right)^2 - \frac{cv}{2} \right\} \right],$$

which is greater than zero as long as

$$\eta = \frac{1}{\sqrt{\tau}} \left[ \delta' z + \sigma \left\{ z' z - \frac{2}{\tau} \left( \delta' z \right)^2 - \frac{cv}{2} \right\} \right],$$

is greater than zero. Let  $f(\eta)$  denote the pdf of  $\eta$ . Then to order  $O(\sigma)$ 

$$P[(M(b) - M(\hat{\beta}_h) > 0] = \int_0^{\square} f(\eta) d\eta.$$

To obtain the pdf  $f(\eta)$  we observe that, to order  $O(\sigma)$ , the characteristic function of  $\eta$  is given by:

$$\psi(t) = \operatorname{E}[\exp(it\eta)]$$

$$= \operatorname{E}[\exp\left\{\frac{it}{\sqrt{\tau}} \left(\delta'z + \sigma z'z - \frac{2\sigma}{\tau}(\delta'z)^2 - \frac{cv}{2}\right)\right\}]$$

$$= \operatorname{E}[\exp\left(it\frac{\delta'z}{\sqrt{\tau}}\right) \left\{1 + \frac{it\sigma}{\sqrt{\tau}} \left(z'z - 2\frac{(\delta z)^2}{\tau} - \frac{cv}{2}\right)\right\}]$$

$$= \exp\left(-\frac{1}{2} t^2\right) \left[1 + \frac{i\sigma}{\sqrt{\tau}} \left\{t(k-2) + t^3 - c\frac{(T-k)}{2} t\right\}\right]$$

Now, utilizing the inversion formula

$$f(\eta) = \frac{1}{2\pi} \int_{-\Box}^{\Box} e^{-it\eta} \psi(t)dt,$$

along with the results that

$$\frac{1}{2\pi}$$
  $\int_{-\Box}^{\Box} \exp(-it\eta - \frac{1}{2} t^2) dt = \phi(\eta),$ 

$$\frac{1}{2\pi} \int_{-\Box}^{\Box} \operatorname{texp}(-it\eta - \frac{1}{2} t^2) dt = -i\eta\phi(\eta),$$

$$\frac{1}{2\pi} \int_{-\Box}^{\Box} t^3 \exp(-it\eta - \frac{1}{2} t^2) dt = -i(3\eta - \eta^3) \phi(\eta),$$

where  $\phi(.)$  denotes the pdf of a standard normal variate, we get the following approximate expression for  $f(\eta)$ :

$$f(\eta) = \phi(\eta) \left[ 1 + \frac{\sigma}{\sqrt{\tau}} \left\{ \eta(k+1) - \eta^3 - \frac{c}{2} \eta(T-k) \right\} \right].$$

Thus, to order  $0(\sigma)$ , we obtain

$$P[M(b) - M(\hat{\beta}_h) > 0]$$

$$= \int_0^{\square} f(\eta) d\eta$$

$$= \frac{1}{2} + \frac{\sigma}{(2\pi\tau)^{1/2}} [(k-1) - \frac{c}{2} (T-k)],$$

which leads to (5).

Following Tran Van Hoa and Chaturvedi (1993), to order  $0(\sigma^4)$ , we can write

$$(\hat{\beta}_{s}-\beta)'X'X(\hat{\beta}_{s}-\beta)-(\hat{\beta}_{h}-\beta)'X'X(\hat{\beta}_{h}-\beta)$$

= 
$$2cc*(1-w)\sigma^5 \frac{v2}{\tau^2} \left[ -\delta'z - \sigma\{z'z - \frac{4}{\tau}(\delta'z)^2 - cv\} \right],$$

which is greater than zero if and only if

$$\gamma = \frac{1}{\sqrt{\tau}} \left[ -\delta' z + \sigma \{ z'z - \frac{4}{\tau} (\delta'z)^2 - cv \} \right],$$

is less than zero.

Now, to order  $\theta(\sigma)$ , the characteristic function of  $\gamma$  is given by:

$$\psi^*(t) = \exp\left(-\frac{1}{2}t^2\right) \left[1 + i\frac{\sigma}{\sqrt{\tau}} \left\{ (k-4) t + 3t^3 - cvt \right\} \right].$$

Hence, using the inversion formula, the pdf of  $\gamma$ , to order  $\theta(\sigma)$ , is given by

$$f^*(\gamma) = \phi(\gamma) \left[ 1 + \frac{\sigma}{\sqrt{\tau}} \left\{ (k+5)\eta - 3\eta^3 - cv\eta \right\} \right].$$

Therefore, to order  $0(\sigma)$ , we have

$$P[M(\stackrel{\wedge}{\beta}_S) - M(\stackrel{\wedge}{\beta}_h) > 0] = P[\gamma < 0]$$

$$= \int_{-\square}^{0} f^{*}(\gamma) d\gamma$$

$$= \frac{1}{2} - \frac{\sigma}{\sqrt{2\pi\tau}} [(k-1) - c(T-k)],$$

which gives the result (6).