Optimal Taxation in a Growth Model with Public Capital Stock and Adjustment Costs

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Abstract

This paper extends Turnovsky (1996a, b) into a dynamic AK growth model with public capital accumulation and adjustment costs, and studies the optimal tax structure between capital and consumption. A particular focus is on whether rejection of the Chamley (1986) proposition obtained in Turnovsky (1996a, b) could be generalized, and how the adjustment cost of public capital would affect the optimal tax structure, between capital and consumption taxes. It finds that, under proper parameter values, there is a unique interior optimal tax structure to support the market equilibrium allocation as the first-best optimum allocation. The interior optimal positive capital tax rate generalizes Turnovsky's result into an economy without congestion in the use of public capital. It also finds that the adjustment cost of public capital reduces *equilibrium* economic growth in transitions and steady state, but it has a zero effect on the first-best *optimal* growth rate. Different from the zero effect of adjustment cost of private capital on optimal taxation in Turnovsky, the adjustment cost of public capital always raises the average tax rate through increasing an optimal consumption tax rate, while its effect on an optimal capital tax rate is ambiguous.

Key words: government capital, Tobin's q, optimal taxation, economic growth
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I. Introduction

This paper studies the optimal tax structure between capital and consumption , in a dynamic growth model with public capital stock in production. Debates about whether income/capital or consumption should be taxed for the welfare of an economy date at least back to Fischer (1937). Early studies on this issue either focus on the efficiency and equity arguments in a static model (Atkinson and Stiglitz,1980), or are based on a neoclassical model with an exogenous economic growth rate (Diamond and Mirrlees, 1971). A large volume of conventional wisdom proposes direct taxation based upon consumption for the efficiency argument. In particular, in a well-known Ramsey model, Chamley (1986) establishes that asymptotically, the optimal taxation on capital, should converge to zero. This result is known as the Chamley proposition, which has motivated many follow-ups supporting its robustness. See the unified work by Atkeson, Chari and Kehoe (1999) that establishes a zero capital tax.

However, neither a static nor a Ramsey framework, is appropriate for addressing the question of optimal tax within a world of ongoing growth. In recent two papers, Turnovsky (1996a, b) extends the endogenous growth models by Romer (1986), Barro (1990) and Rebelo (1991), to revisit the optimal tax structure between capital and consumption. In his AK models with public capital entering as a factor of production, Turnovsky (1996a, b) finds that, under zero congestion in using public capital, the Chamley proposition still survives when public expenditure is optimally set; on the other hand, under positive congestion in using public capital, it is necessary to tax both capital and consumption, ruling out zero capital taxation as an optimal policy.[1] Unlike existing endogenous growth models with taxation (e.g., Rebelo, 1991), Turnovsky (1996a, b) allows for the interdependence between government's revenue and expenditure decisions in the determination of overall optimal fiscal policy. As a result, the motivation for a government to tax is well established. However, as it is public investment that affects the private sectors' production,

[1] Turnovsky (1996b) also introduces the adjustment costs of private capital into his model, and finds that the adjustment costs do not affect the optimal tax structure, when public expenditure is optimally set.

Turnovsky (1996a, b) only obtains long-run growth, like that of Rebelo (1991). Since Chamley (1986) is a model with transitional dynamics, the results obtained in Turnovsky (1996a, b) may appear to be special. In order to examine the robustness of Turnovsky's results in a dynamic framework, it is necessary to allow public investment to accumulate capital stock.[2]

The purpose of this paper is to revisit the optimal tax structure between capital and consumption in a dynamic AK model with public capital. A particular focus is on whether the rejection of the Chamley (1986) proposition in Turnovsky, can be generalized into a dynamic framework, without resorting to a congestion assumption. This model differentiates itself from conventional wisdom in that it (i) allows public investment to accumulate capital and (ii) considers the adjustment costs of the accumulation. First, while the former feature renders an otherwise long-run AK model to generate transitional dynamics, it also makes the dependence on congestion in rejecting the Chamley proposition unnecessary, since capital taxation is like a user's fee, given positive initial public capital stock. Secondly, the latter feature links the "Tobin q" theory of investment to public investment. As the government optimally determines public investment in order to accumulate capital, it is natural to introduce adjustment costs.[3] Recent investment theory motivates the derivation of the Tobin q theory from convex adjustment costs of capital accumulation (e.g., Hayashi,1982). The adjustment cost approach has been applied extensively to study issues pertaining to tax policies in a Ramsey model. This approach has been adopted in an endogenous growth model by Turnovsky (1996b), which finds that the adjustment costs of private capital do not affect optimal tax structure when government

[2] Other ways are also possible to generate growth dynamics; e.g., open economies with limited access to the world financial market. Allowing public investment to accumulate capital stock has recently gained much attention in endogenous growth models (e.g., Futagami, Morita and Shibata, 1993, Glomm and Ravikumar, 1994, and Turnovsky,1997a, b) and a Ramsey model (e.g., Fisher and Turnovsky, 1998).

[3] To our knowledge, no existing Ramsey and endogenous growth models have considered the adjustment costs of public capital, except for Chatterjee, Sakoulis and Turnovsky (2001) in a model with international capital transfers. Using a numerical method, their paper studies how temporary and permanent international transfers affect economic growth and transitional dynamics. Optimal tax structure is not emphasized.

expenditure is optimally set. It is interesting to examine how the adjustment costs of public capital affect optimal allocations and optimal tax structure, as opposed to the adjustment costs of private capital.

In this model, the government first announces a program of tax rates and expenditures. We study a command economy in which the government adopts policies without considering the representative agent's responses, and a market economy in which the government takes the representative agent's responses into account when setting optimal policies. The optimal tax structure in a market economy is known as the second-best optimum, as private sectors ignore the effects of their behavior on others, albeit the government has taken this aspect into account in designing the optimal tax structure. We characterize market equilibrium in both steady state and transitional dynamics, and examine whether it is possible to design a tax structure, in order for the steady-state market equilibrium allocations to achieve the first-best optimal allocations.

The main findings are briefly stated as follows. First, under proper parameter values, there is a unique interior tax structure for capital and consumption so that the market equilibrium can support the first-best optimal allocation. The interior optimal capital tax rates indicate that the rejection of Chamley's zero capital taxation by Turnovsky (1996a, b) is generalized within an economy without congestion, when using government services. Secondly, although the adjustment cost of public capital reduces *equilibrium* economic growth in transitions and steady state, it neither affects the first-best *optimal* public to private capital ratios, nor the first-best *optimal* growth. Finally, the adjustment cost raises optimal average tax rates through increasing optimal consumption tax rates, but with an ambiguous effect on optimal capital tax rates. These second and third results stand in sharp contrast to those in Turnovsky (1996b), where the adjustment cost of *private capital* affects optimal public to private capital ratio, but not optimal tax structure.

The structure of the paper is organized as follows. In Section II, a basic model is set up. While a centrally-planned economy is studied in Section III, a market economy is investigated in Section IV. Section V envisages the second-best and the first-best optimal tax structure. Finally, Section V is the conclusion.

II A Basic Model

Our basic model builds on Barro (1990), Futagami, Morita and Shibata (1993) and Turnovsky (1996a, b). Consider an economy populated by a continuum of representative household agents who live infinitely. There is no population growth, and the population size is normalized to a unity. There exists a continuum of representative firms, each of which is endowed with a production technology with households owning the shares. It follows that the economy is a world of representative households-producers. Additionally, there is the government.

A representative household derives lifetime utilities from the following isoelastic utility function:

$$U = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt, \quad \rho > 0, \ -\infty < 1-\sigma < 1,$$

in which $c(t)$ is the instantaneous private consumption expenditure in t. Parameter ρ is the instantaneous time-preference rate, and σ is the reciprocal of the intertemporal elasticity of substitution for consumption. Restriction $1-\sigma < 1$ assures a strictly concave function.

The productivity of a production technology is enhanced externally by public expenditure, as in Barro (1990) and Turnovsky (1996a, b). Unlike these authors, it is the stock of public infrastructures, and not the flow, that affects the production technology in our model. This setup is analogous to that in Futagami, Morita and Shibata (1993), and warrants the fruits of public investment to emerge in the future in a fashion consistent with private capital, as opposed to instantaneous benefits of both public and private consumption. Both public and private capital stocks enter the following production technology:

$$y(t) = Ak(t)^{\beta} g(t)^{1-\beta},\ A>0,\ 0<\beta<1, \qquad \textbf{(1)}$$

in which *y(t)* is the instantaneous output per capita in *t*, *k(t)* is the private capital stock per capita in *t*, and *g(t)* is the public capital stock per capita in *t*. Parameter 1-β captures the degree of externality to which public infrastructure affects private production, and $A>0$ summarizes the productivity level. For simplification, the production function does not introduce any congestion from using public capital, different from Turnovsky (1996a, b).[4] The Cobb-Douglas form ensures that the profit-maximization problem faced by each firm is concave and well-defined. Without loss of generality, we assume a zero depreciation rate for both public and private capital stock. Each firm is competitive in the goods and the input markets.

While public infrastructures are accumulated from public investment:

$$\dot{g} = I_g(t), \qquad \textbf{(2a)}$$

the accumulation requires the following convex adjustment costs:

$$\Phi(I_g(t),\ g(t)) = I_g(t)\left(1+\varphi\frac{I_g(t)}{2g(t)}\right),\ \varphi>0, \qquad \textbf{(2b)}$$

in which $I_g(t)$ is the public investment flow in *t*. As the government provides infrastructures free of charge, the following government budget constraints must be satisfied:

$$I_g(t)\left(1+\varphi\frac{I_g(t)}{2g(t)}\right) = T(t), \qquad \textbf{(3a)}$$

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[4] Congestion in using public capital can be easily introduced into our model by setting $y(t) = Ak(t)^{\beta}\{g(t)[k(t)/\bar{k}]^{\sigma}\}^{1-\beta},\ 0\leq\sigma\leq 1,$ where $\bar{k}$ is the economy-wide average private capital, and σ is the degree of congestion. The results derived under no congestion σ=0 in our model carry over for $0<\sigma\leq 1$. Therefore, it simplifies the analysis not to introduce the degree of congestion.

$$T(t) \ = \ \tau_k A k(t)^{\beta} g(t)^{1-\beta} + \tau_c c(t), \tag{3b}$$

in which $T(t)$ is total tax revenues in t, with τ_k as capital tax rate, and τ_c as consumption tax rate.[5] While (3a) is the government budget constant, (3b) describes the sources of tax revenues.

Although public investment leads to public capital accumulation benefitting private sectors, the government taxation accompanies a drop in unspent households' income, reducing private capital formation. Under a zero capital depreciation assumption, a household's budget constraints become:

$$\dot{k} = A k(t)^{\beta} g(t)^{1-\beta} - \ c(t) \ - \ T(t), \tag{4}$$

which also describes how private capital stock evolves.

III. A Command Economy

In a centrally-planned economy, the government announces a program of expenditures and taxes without considering the responses of the representative agent. To solve the central planner's optimization problem, define the following present-value Hamiltonian equation:

$$H \equiv e^{-\rho t}\left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \lambda(t)\dot{k} + \mu'(t)\dot{g} \right),$$

where λ and μ' are the shadow price of private and public capital stock, respectively. Together with (2a), (3a) and (4), the first order conditions are:

[5] We follow Turnovsky (1996a, equations 5 and 15b) to label τ_k as a capital tax rate because capital is the only private factor of production in our model.

$$-\sigma \frac{\dot{c}(t)}{c(t)} = \frac{\dot{\lambda}}{\lambda(t)}, \qquad \textbf{(5a)}$$

$$1 + \varphi \frac{I_g(t)}{g(t)} = \frac{\mu'(t)}{\lambda(t)}, \qquad \textbf{(5b)}$$

$$A\beta \left(\frac{g(t)}{k(t)} \right)^{1-\beta} = \rho - \frac{\dot{\lambda}}{\lambda(t)}, \qquad \textbf{(5c)}$$

$$A(1-\beta) \left(\frac{g(t)}{k(t)} \right)^{-\beta} = \rho - \frac{\dot{\mu}'}{\mu'(t)}, \qquad \textbf{(5d)}$$

$$Lim_{t \to \infty} \ \lambda(t)k(t)e^{-\rho t} = 0. \qquad \textbf{(5e)}$$

While (5a) equates the instantaneous marginal utility of consumption to private capital gains, (5b) equates the instantaneous marginal return of private capital to the shadow price of public capital, relative to the shadow price of private capital. Conditions (5c)-(5d) are Euler equations, describing two non-arbitrage conditions: they equate the marginal productivity of private capital and of public capital, respectively, to the time-preference rate, both adjusted for capital gains. Finally, (5e) is the transversality condition that avoids $k(t)$ from growing too fast.[6] Simple algebra leads (5b) to:

$$\frac{I_g(t)}{g(t)} \equiv \frac{\dot{g}}{g(t)} = \frac{1}{\varphi} \left(\frac{\mu'(t)}{\lambda(t)} - 1 \right). \qquad \textbf{(6)}$$

Relationship (6) is a Tobin-q version of public investment, having positive public investment if, using the price of new output as numeraire, the shadow price of public capital is greater than 1.

Denote $x(t) \equiv \frac{c(t)}{k(t)}$, $z(t) \equiv \frac{g(t)}{k(t)}$, and $\mu(t) \equiv \frac{\mu'(t)}{\lambda(t)}$. Then, taking differences between (5c) and (5d), (5a) and (4), and (2a) and (4), together with (6), yields, respectively:

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[6] The transversality condition $Lim_{t \to \infty} \ \mu'(t)g(t)e^{-\rho t} = 0$ is automatically met under (5e), as $g(t)$ is limited by the output and thus $k(t)$.

$$\frac{\dot{\mu}}{\mu(t)} \equiv \frac{\dot{\mu}'}{\mu'(t)} - \frac{\dot{\lambda}}{\lambda(t)} = A\beta z(t)^{1-\beta} - A(1-\beta)z(t)^{\beta}, \tag{7a}$$

$$\frac{\dot{x}}{x(t)} \equiv \frac{\dot{c}}{c(t)} - \frac{\dot{k}}{k(t)} = -\frac{\rho}{\sigma} + x(t) + \frac{\mu(t)-1}{\varphi}\left(1+\frac{\mu(t)-1}{2}\right)z(t) - A\left(1-\frac{\beta}{\sigma}\right)z(t)^{1-\beta}, \tag{7b}$$

$$\frac{\dot{z}}{z(t)} \equiv \frac{\dot{g}}{g(t)} - \frac{\dot{k}}{k(t)} = \frac{-1}{\varphi} + x(t) + \frac{\mu(t)}{\varphi} + \frac{\mu(t)-1}{\varphi}\left(1+\frac{\mu(t)-1}{2}\right)z(t) - Az(t)^{1-\beta}. \tag{7c}$$

In steady state $\dot{\mu} = \dot{x} = \dot{z} = 0,$ and therefore the socially optimal steady-state values of μ, x, and z are constant over time. From (7a)-(7c), these steady-state values are derived as:

$$\begin{cases} \mu^* = 1 + \varphi\left[\dfrac{A\beta}{\sigma}\left(\dfrac{1-\beta}{\beta}\right)^{1-\beta} - \dfrac{\rho}{\sigma}\right], \\ z^* = \dfrac{1-\beta}{\beta}, \\ x^* = \dfrac{\rho}{\sigma} + A\left(1-\dfrac{\beta}{\sigma}\right)z*^{1-\beta} - \dfrac{\mu^*-1}{\varphi}\left(1+\dfrac{\mu^*-1}{\varphi}\right)z*. \end{cases} \tag{8a}$$

Steady-state $z* \equiv \frac{1-\beta}{\beta}$ implies that the government chooses public investment, so that the optimal public to private capital ratio equals the ratio between the marginal productivity of public capital to that of private capital. Under this optimal plan, the economic growth rate is:

$$\gamma* \equiv \frac{I*}{k*} = \frac{I_g*}{g*} = \frac{\mu*-1}{\varphi} = \frac{1}{\sigma}\left[A\beta\left(\frac{1-\beta}{\beta}\right)^{1-\beta} - \rho\right]. \tag{8b}$$

In order to guarantee a positive growth rate with a bounded lifetime utility, consider:

Condition PB: (positive growth and bounded utility) $(1-\sigma)A\beta\left(\frac{1-\beta}{\beta}\right)^{1-\beta} < \rho < A\beta\left(\frac{1-\beta}{\beta}\right)^{1-\beta}$.

While the first inequality in Condition PB makes sure that utilities are bounded, the second inequality guarantees a positive economic growth rate. When productivity parameter A is large enough, the second inequality can be met. When one minus the reciprocal of the intertemporal elasticity of substitution (1-σ) is negative, the first inequality is automatically satisfied; when (1-σ) is positive, the first inequality is more likely to meet if the time-preference rate is high.

It is clear from (8a)-(8b) that the adjustment costs of public capital affect the shadow price of public capital, but not the socially optimal allocations (i.e., optimal public to private capital ratio z^*, optimal consumption to private capital ratio x^*,[7] and optimal economic growth γ^*). Nevertheless, in order to implement the socially optimal allocation, the adjustment costs raise the average tax rate, according to (3a):

$$\tau^* \equiv \frac{T(t)}{y(t)} = \frac{1}{A\sigma}\left[A\beta\left(\frac{1-\beta}{\beta}\right)^{1-\beta} - \rho\right]\left\{1+\frac{\varphi}{2\sigma}\left[A\beta\left(\frac{1-\beta}{\beta}\right)^{1-\beta} - \rho\right]\right\}\left(\frac{1-\beta}{\beta}\right)^{\beta} > 0. \qquad \textbf{(9)}$$

Intuitively, the adjustment costs of public capital in our model reflect the inefficiency of public capital accumulation *vis-à-vis* private capital accumulation. As the government optimally maintains a fixed public to private capital ratio in order to obtain an optimal economic growth rate and an optimal consumption to private capital ratio, it uses different tax rates to cover different adjustment costs. In other words, the government accommodates the inefficiency in public capital accumulation with higher tax rates. Since the tax rate increases in public capital accumulation, the shadow price of public capital increases. Whether capital or consumption tax rates, or both, increase in adjustment costs, will be answered in Section V.

The above results may be compared with Turnovsky (1996b). Turnovsky (1996b) is a model where public capital does not accumulate. When the government expenditure is optimally set, wee find that *the*

[7] When we substitute μ^* into x^* in (8a), φ disappears from x^*.

adjustment costs of private capital affect socially optimal economic growth without having any effects upon tax rates. In our model, the government expenditure is optimally set, but *the adjustment costs of public capital* affect only the tax rates, without delivering any impacts upon economic growth.

Proposition 1. *Under Condition PB, there exists a unique first-best optimal allocation in the steady state. The adjustment costs of public capital raise tax rates and the shadow price of public capital.*

IV A Market Economy

We now turn to a market economy. In a market economy, the government announces a program of tax rates and expenditures in the initial period, taking into consideration the private agents' responses. The representative agent is perfectly foresighted. Given tax rates, public expenditures and budget constraints (4) and (3b), the representative agent chooses a program of savings and consumption, in order to maximize his utility. The model is solved in a backward fashion.

First, let ν be the shadow price of private capital, and define a Hamiltonian equation:

$$H \equiv e^{-\rho t}\left(\frac{c(t)^{1-\sigma}}{1-\sigma} + \nu(t)\dot{k} \right).$$

The necessary conditions of optimization leads to the following familiar relationship:

$$\frac{\dot{c}}{c(t)} = \frac{(1-\tau_k)A\beta z^{1-\beta} - \rho}{\sigma}. \qquad \textbf{(10)}$$

Next, the market economy should be in equilibrium. An equilibrium is a tuple $\{x(t), z(t), \frac{y(t)}{k(t)}, \frac{\dot{c}}{c(t)}, \frac{T(t)}{y(t)}, \frac{I_g(t)}{g(t)}\}$, solved by production technology (1), laws of motion of capital (2a) and (4), government budget constraints (3a)-(3b) and households' optimization (10), given tax rates. A steady-state market equilibrium is a balanced-growth path, with all variables in the above tuple constant over time.

IV-1. Steady-State Equilibrium

To determine a steady-state equilibrium, we transform the six-equation economic system into a planar system. Subtracting (10) and (2a), respectively, from (4), together with (3a)-(3b),[8] generates:

$$\frac{\dot{x}}{x(t)} = -\frac{\rho}{\sigma}+(1+\tau_c)x^{**} -(1-\tau_k)A\left(1-\frac{\beta}{\sigma}\right)z^{**1-\beta}=0, \qquad \textbf{(11a)}$$

$$\frac{\dot{z}}{z(t)} = -\frac{1}{\varphi}+(1+\tau_c)x^{**}+\frac{1}{\varphi}[1+2\varphi(\tau_k A z^{**-\beta}+\tau_c x^{**}z^{**-1})]^{1/2}-(1-\tau_k)Az^{**1-\beta}=0. \qquad \textbf{(11b)}$$

While Locus $\dot{z}=0$ starts from the constant $z_0>0$, when $x=0$ at an upward slope,[9] Locus $\dot{x}=0$ starts from constant $\frac{\rho}{\sigma(1+\tau_c)}>0$ when $z=0$, with a positive (negative) slope if the product of a private capital share in production β and the intertemporal elasticity of substitution 1/σ is below (above) 1. When Locus $\dot{x}=0$ is negatively sloping, it intersects z axis at $\frac{\rho}{(1-\tau_k)A(\sigma-\beta)}>0$. Although a logarithmic utility form guarantees $\beta/\sigma<1$, other forms cannot rule out the possibility of $\beta/\sigma>1$. Figure 1 illustrates Locus $\dot{x}=0$ for both cases. As Loci $\dot{z}=0$ and $\dot{x}=0$ intersect once, regardless of their slopes, there is a unique steady state.[10] Therefore, the planar system uniquely determines steady-state equilibrium values $x^{**}=\left(\frac{c(t)}{k(t)}\right)^{**}$ and $z^{**}=\left(\frac{g(t)}{k(t)}\right)^{**}$ as functions of adjustment costs and tax rates (ϕ, τ_k and τ_c).[11] After obtaining x^{**} and z^{**}, we substitute into

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[8] (3a)-(3b) imply $\frac{I_g}{g}=\frac{1}{\varphi}(-1+[1+2\varphi(\tau_k A z^{-\beta}+\tau_c x z^{-1})]^{\frac{1}{2}})$.

[9] z_0 satisfies $[1+2\varphi\tau_k A z^{-\beta}]^{1/2}-\varphi(1-\tau_k)Az^{1-\beta}=1$.

[10] When both Loci $\dot{z}=0$ and $\dot{x}=0$ are positively slopping, Locus $\dot{z}=0$ is steeper than Locus $\dot{x}=0$, in order to guarantee a saddle path. This will be verified in Section IV-2 below.

[11] In what follows, we use two asterisks to denote the equilibrium and optimum in a market economy, as opposed to an asterisk to denote the optimum in a centrally-planed economy.

(1), (10) and (3b) and (2a) to obtain the other four steady-state equilibrium values, $\left(\frac{y(t)}{k(t)}\right)^{**}$, $\left(\frac{\dot{c}}{c(t)}\right)^{**}$, $\left(\frac{T(t)}{y(t)}\right)^{**}$, and $\left(\frac{I_g(t)}{g(t)}\right)^{**}$, respectively. Therefore, all six endogenous variables along a BGP are obtained.

[Insert Figure 1 here]

Substituting z^{**} into (10) yields a constant economic growth rate along a BGP:

$$\gamma^{**}(z^{**}) = \frac{(1-\tau_k)A\beta z^{**^{1-\beta}} - \rho}{\sigma}. \tag{12}$$

Different from the socially optimal economic growth rate γ^* in (8b), that is independent of public capital adjustment costs, an equilibrium economic growth rate depends upon the capital adjustment costs, through effects upon an equilibrium public to private capital ratio.

Let [a, b] be the feasible set for z. Then, in order to attain positive growth and bounded lifetime utility, it suffices to modify Condition PB as:

Condition P′B′: $(1-\sigma)(1-\tau_k)A\beta a^{1-\beta} < \rho < (1-\tau_k)A\beta b^{1-\beta}$.

Condition P′B′ is automatically met under Condition PB, as $z^* \equiv (1\text{-}\beta)/\beta \in [a, b]$.

IV-2. Transitional Dynamics

Linearizing planar system (11a)-(11b), around the unique steady state, yields:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x(t) - x^{**} \\ z(t) - z^{**} \end{pmatrix}, \tag{13}$$

where $a_{11} = (1+\tau_c)x^{**} > 0$, $a_{12} = -(1-\beta)(1-\tau_k)A(1-\frac{\beta}{\sigma})\frac{x^{**}}{z^{**^\beta}} \lessgtr 0 \; if \; \frac{\beta}{\sigma} \lessgtr 1$, $a_{21} = [(1+\tau_c) + \frac{\tau_c}{\Phi}\frac{1}{z^{**}}]z^{**} > 0$,
$a_{22} = -[\frac{\beta\tau_k A}{\Phi}\frac{1}{z^{**^{\beta+1}}} + (1-\tau_k)A(1-\beta)\frac{1}{z^{**^\beta}}]z^{**} < 0$, and $\Phi \equiv [1 + 2\varphi(\frac{\tau_k A}{z^{**^\beta}} + \frac{\tau_c x^{**}}{z^{**}})]^{1/2} \geq 1$.

In order to obtain a unique saddle path, it requires:

$$\textit{Product of two roots in } (13) \equiv \underset{(+)(-)}{a_{11}a_{22}} - \underset{(+)(?)}{a_{21}a_{12}} < 0,$$

which implies that the slope of $\dot{z}=0$ must be larger than that of $\dot{x}=0$:

$$0 < \left.\frac{dx}{az}\right|_{\dot{z}=0} = -\frac{a_{22}}{a_{21}} > \left.\frac{dx}{az}\right|_{\dot{x}=0} = -\frac{a_{12}}{a_{11}} \ \substack{> \\ <} \ 0 \quad if \quad \frac{\beta}{\sigma} \ \substack{< \\ >} \ 1.$$

When the product of private capital share and the intertemporal elasticity of substitution is smaller than 1 (i.e., β/σ<1), both Loci $\dot{z}=0$ and $\dot{x}=0$ slope positively, with the former locus being steeper than the latter, to guarantee a saddle path. Under this case, the unique saddle path is positively slopping, as illustrated in upper Figure 1. On the other hand, when the product is larger than 1 (i.e., β/σ>1), while Locus $\dot{z}=0$ is still positively slopping, Locus $\dot{x}=0$ becomes negatively slopping. Under this case, the unique saddle path is negatively slopping, as illustrated in lower Figure 1.

Under both cases β/σ<1 and β/σ>1, starting from an initial state in Figure 1, say $z(0)<z^{**}$, the economy's equilibrium moves along a unique saddle path toward the steady state. In both cases, public to private capital ratios monotonically increase in transition, until a steady state is reached. As a result, the economic growth rate also monotonically increases over time until a steady state is reached. However, the consumption to private capital ratio, starts from below its steady-state level and increases monotonically in transition to the steady-state level, when the intertemporal elasticity of substitution is small (i.e., β/σ<1); but it starts from above its steady-state level and decreases monotonically in transition to the steady state, when the intertemporal elasticity of substitution is large (i.e., β/σ>1).

Summarizing the transitional dynamics and steady-state equilibrium, we obtain:

Proposition 2. *Under Condition P B′, there exists a unique saddle path leading the market equilibrium* *toward a unique steady state. Starting from a small public to private capital ratio, public to private capital ratios and economic growth both increase, while consumption to private capital ratios increase (decrease), when the product of private capital shares and intertemporal elasticity is smaller (larger) than one.*

IV-3. Equilibrium Properties

We now examine the economy's equilibrium properties. In particular, we are interested in the effects of a unit cot of public capital adjustment and two tax rates, upon consumption to private capital ratios, public to private capital ratios, and economic growth.[12]

3-1. Higher unit adjustment cost

First, a higher unit adjustment cost ϕ shifts Locus $\dot{z}=0$ leftward without affecting Locus $\dot{x}=0$ (see Figure 2). Intuitively, a higher unit adjustment cost of public capital discourages public capital accumulation, thereby lowering public to private capital ratios in a steady state and in a transition to it. This in turn leads economic growth to decrease monotonically until a new steady state. However, the effect upon consumption to private capital ratios depends on the product of a private capital share and intertemporal elasticity. When the product is smaller than one, both consumption and private capital to public ratios need to be positively correlated to maintain a steady-state consumption to private capital ratio. Under this case, smaller public to private capital ratios in steady state result in small consumption to private capital ratios, starting with an instantaneous drop in consumption to private capital ratios so that the equilibrium path moves along a new saddle path (see upper Figure 2). On the other hand, when the product is larger than 1, consumption to private capital ratios are negatively associated with public to private capital ratios to maintain steady-state consumption to private capital ratios. Under this case, smaller public to private capital ratios in a new steady state result in larger consumption to private capital ratios (see lower Figure 2).

[12] Mathematical derivation for the following comparative-statics effects upon public to private capital ratios, and consumption to private capital ratios in steady state, is delegated in Appendix.

Although the above negative equilibrium growth effect of higher adjustment costs is the same as in Turnovsky (1996b), the channel is different. While the negative growth effect of the capital's adjustment costs is direct through the reduction of *private* capital accumulation in Turnovsky (1996b), it is via the reduction of *public* capital accumulation, which in turn adversely affects private capital in our study.

[Insert Figure 2 here]

3-2. Higher capital tax rate

Next, a higher capital tax rate rotates Locus $\dot{z}=0$ rightward. Locus $\dot{x}=0$ rotates downward when the product of a private capital share and intertemporal elasticity is smaller than one (i.e., $\beta/\sigma<1$; upper Figure 3), but rotates upward when the product is larger than one (i.e., $\beta/\sigma>1$; lower Figure 3). Under Case $\beta/\sigma<1$, a higher capital tax rate may decrease (Points B and C) or increase (Point A) consumption to private capital ratios instantaneously, depending on whether Locus $\dot{x}=0$ is, or is not, sensitive to capital taxes. Over time, consumption to private capital ratios and public to private capital ratios may decrease (Path CE_3), or increase (Paths AE_1 and BE_2). As a result, economic growth increases in transitions until a new steady state in the latter situation (along Paths AE_1 and BE_2), but decreases in the former context (along Path CE_3). On the other hand, under Case $\beta/\sigma>1$, consumption to private capital ratios increase instantaneously, followed by a rebound in transition until new steady state E' is reached. The consumption to capital ratio at the new steady state E' may be higher or lower than the ratio at original steady state E. However, public to private capital ratios must increase monotonically until new steady state E'. Consequently, economic growth increases.

Different from existing literature on public policy and growth where a higher capital tax has a direct negative growth effect in our model (e.g., Rebelo, 1991) , a larger capital tax rate may have a positive growth effect in our model because of public investment. Although this result is in line with Barro (1990) and Turnovsky (1996a, b), the outcome of the net growth effect depending upon the product of private capital share and intertemporal elasticity, differentiates ours from theirs. Particularly one difference is that higher capital tax rates always lead to larger growth rates in our model, when the product is larger than 1. This result emerges because public investment can accumulate capital stock. Intuitively, when capital tax rates are raised, consumption increases instantaneously because of high elasticity of substitution between savings and consumption. Higher capital tax rates also increase tax revenues and government expenditure. Under a high private capital share, the marginal product of private capital is high for a given public capital stock, and thus, public capital is more productive. Under conditions that the product of a private capital share and elasticity of substitution is greater than 1, the positive growth effect from a high marginal product dominates the negative growth effect from the substitution, and thereby raises economic growth.

[Insert Figure 3 here]

3-3 Higher consumption tax rate

Finally, a higher consumption tax rate rotates Locus $\dot{z}=0$ downward, and shifts Locus $\dot{x}=0$ downward (Figure 4). As a result, consumption drops and savings increase instantaneously. As higher consumption tax revenues facilitate public investment, public capital accumulates over time, increasing economic growth monotonically until new steady state E′ is reached. If the product of a private capital share and intertemporal elasticity is larger than 1, consumption to private capital ratios decease monotonically in transitions to a new steady state (see lower Figure 4). However, if the product of a private capital share and intertemporal elasticity is smaller than 1, consumption to private capital ratios increase monotonically but, under proper parameter values, the ratio at the new steady-state is lower than the original steady-state level x^{**}.[13] We should mention that, different from the reduction in Turnovsky (1996a, b), after an immediate drop, our consumption to private capital ratios increase in transitions to a steady state, when the product of a private capital share and intertemporal elasticity is smaller than 1. The reason lies in the accumulation of public capital stock, where after an immediate drop in consumption in response to a consumption tax rate, a positive growth effect increases consumption more than private capital in transitions, due to small

[13] The requirement is $x^{**2} < \frac{\tau_k}{1-\beta}\frac{\beta}{1-\tau_k}$, which is easy to meet as x is much smaller than 1, while τ_k is about 1-β.

intertemporal elasticity and a small private capital share. Finally, the positive growth effect of consumption taxes is mitigated both in transitions and steady state, when public capital's adjustment costs are higher. Although Turnovsky (1996) has derived a similar result for higher private capital's adjustment costs in steady state, he has not obtained a result in transitions.

[Insert Figure 4 here]

To summarize the above properties, we obtain:

Proposition 3. *The equilibrium possesses the following properties:*

(i) *higher adjustment costs of public capital always reduce public to private capital ratios and economic growth, and reduce (increase) consumption to private capital ratios when the product of a private capital share and intertemporal elasticity is smaller (larger) than 1;*

(ii) *higher capital tax rates have an ambiguous (positive) effect on consumption to private capital ratios, public to private capital ratios, and economic growth when the product of a private capital share and intertemporal elasticity is smaller (larger) than 1.*

(iii) *higher consumption tax rates always increase public to private capital ratio and economic growth, and reduce consumption to private capital ratios when the product of a private capital share and intertemporal elasticity is larger than 1.*

V. Optimal Tax Structure in a Market Economy

The government sets optimal tax structure before the transactions in a market. It does so by taking into account the representative agent's responses in steady state. Under a given set of tax rates, the representative agent's behavior implies $\frac{c(t)}{k(t)} = x^{**}(\tau_k, \tau_c)$ in steady state, determined by (11a)-(11b). Therefore, $c(t) = k(0) e^{\gamma^{**}} x^{**}(\tau_k, \tau_c)$, where $k(0)$ is given, and $\gamma^{**}(z^{**}(\tau_k, \tau_c)) = \frac{(1-\tau_k) A \beta z^{**}(\tau_k, \tau_c)^{1-\beta} - \rho}{\sigma}$ is determined by (12). Substituting $c(t)$ into the representative household's lifetime utility yields:

$$U \equiv \frac{k(0)^{1-\sigma}}{1-\sigma}\,\frac{x^{**}(\tau_y,\tau_c)^{1-\sigma}}{\rho-(1-\sigma)\gamma^{**}(z^{**}(\tau_y,\tau_c))}. \tag{14}$$

V-1. Second-Best Taxes

In second-best optimization, the government chooses a set of optimal tax structure $\{\tau_k, \tau_c\}$ to maximize the above lifetime utility, taking into account households' best responses represented by (11a)-(11b) and (12). The optimal conditions are:

$$\frac{dU}{d\tau_k} \equiv \frac{k(0)^{1-\sigma}x^{**1-\sigma}}{[\rho-(1-\sigma)\gamma^{**}(z^{**})]^2\tau_k}\{[\gamma^{**}(\tau_k^{**},\tau_c^{**})+\frac{\rho}{\sigma}]\,[\underset{\substack{(-)\\(-)}}{\frac{-\tau_k}{1-\tau_k}}+(1-\beta)\underset{\substack{(?)\\(+)}}{\xi_{z\tau_k}}]+\underset{\substack{(?)\\(?)}}{\xi_{x\tau_k}}\}\le 0, \quad =0 \; if \; \tau_k>0. \quad \substack{(if\;\beta/\sigma<1)\\(if\;\beta/\sigma>1)} \tag{15a}$$

$$\frac{dU}{d\tau_c} \equiv \frac{k(0)^{1-\sigma}x^{**1-\sigma}}{[\rho-(1-\sigma)\gamma^{**}(z^{**})]^2\tau_c}\{[\gamma^{**}(\tau_k^{**},\tau_c^{**})+\frac{\rho}{\sigma}](1-\beta)\underset{\substack{(+)\\(+)}}{\xi_{z\tau_c}}+\underset{\substack{(?)\\(-)}}{\xi_{x\tau_c}}\}\le 0, \quad =0 \; if \; \tau_c>0. \quad \substack{(if\;\beta/\sigma<1)\\(if\;\beta/\sigma>1)} \tag{15b}$$

where, although its sign could be positive or negative, $\xi_{h\theta}$ is called the elasticity of h with respect to a policy variable θ, $h = x$ and z, and $\theta = \tau_k$ and τ_c.

Optimal tax rates τ_y^{**} and τ_c^{**} are simultaneously determined by (15a) and (15b). In characterizing the optimal tax structure, it is clear that optimal tax rates depend on both the elasticity of public to private capital ratios, and that of consumption to private capital ratios with respect to tax rates, which in turn rely on how a representative household behaves in response to a tax rate, summarized in (11a)-(11b). Depending upon the product of a private capital share and intertemporal elasticity, the household's responses vary. We characterize the second-best tax structure according to the size of the product.

1-1. Case $\beta/\sigma < 1$ (small private capital share and/or intertemporal elasticity)

Under this case, public to private capital ratios respond positively to consumption tax rates ($\xi_{z\tau c}$>0). Consumption to private capital ratios react ambiguously to consumption tax rates, to the extent that the direct negative effects dominate the indirect positive effect, $\xi_{x\tau c}$<0. Under this condition, (15b) can be zero, implying an interior optimal consumption tax rate.

The effect of capital tax rates are more complicated. It includes an unambiguous direct negative effect, but the effect on the public to private capital and the consumption to private capital ratios are both ambiguous. We use upper Figure 3 to analyze. Consider the situation, where a higher capital tax rate shifts Locus $\dot{z}=0$ upward (so that both $\xi_{z\tau k}$>0 and $\xi_{x\tau k}$>0), or downward, and generates a new steady state with a higher public to private capital ratio like E_2 (so that $\xi_{z\tau k}$>0 and $\xi_{x\tau k}$<0). Then, (15a) can become zero, resulting in an interior capital tax rate. Therefore, (15a) and (15b) simultaneously determine a set of optimal interior tax rates, τ_k^{**}>0 and τ_c^{**}>0. Alternatively, if a higher capital tax rate shifts Locus $\dot{z}=0$ downward and generates smaller public to private capital ratios like the new steady state E_3 (so that $\xi_{z\tau k}$<0 and $\xi_{x\tau k}$<0), then (15a) is negative, implying a zero optimal capital tax rate (τ_k^{**}=0). Therefore, a positive consumption tax rate τ_c^{**}>0 is set according to (15b).

1-2. Case β/σ >1 (large private capital share and/or intertemporal elasticity)

Under this case, (15b) can become zero and yields an interior optimal consumption tax rate. As for (15a), although $\xi_{x\tau k}$ remains ambiguous, there is already a negative term ($-\tau_y/(1-\tau_y)$) and a positive term ($\xi_{z\tau y}$). Thus, (15a) can become zero, bringing forth an optimal interior capital tax rate. Therefore, under a large private capital share and/or large intertemporal elasticity, the interior optimal tax rates τ_y^{**}>0 and τ_c^{**}>0 are simultaneously determined by (15a)-(15b).

Proposition 4. *When the product of a private capital share and intertemporal elasticity is smaller than one, the second-best consumption tax rate is positive, and the capital tax rate is also positive, if steady-state public to private capital ratios are higher in response to a capital tax rate; when the product is larger than* *one, the second-best capital and consumption tax rate are both positive.*

V-2. First-Best Taxes

As the above optimal tax structure takes into account the responses in the private sector which do not internalize the productivity of public capital, it is only second-best optimum. We now examine to see whether it is possible for the government to design a set of tax rates in a market economy, to replicate the first-best optimal allocation. Technically, it suffices if steady-state market equilibrium allocations achieve the first-best utility level. This requires equilibrium time-series value $c(t)$, to be the same as its socially optimal value. Given initial $k(0)$, indirect utility (14) demands the satisfaction of two conditions in order to attain the first-best optimum: (i) the economic growth rate under a market equilibrium must be the same as that under the first-best optimum (Condition $\gamma^* = \gamma^{**}$), and (ii) the consumption to private capital ratio under a market equilibrium must be the same as that under the first-best optimum (Condition $x^* = x^{**}$). Using x^* and z^* from (8a), γ^* from (8b) and γ^{**} from (12), Condition $\gamma^* = \gamma^{**}$ is rewritten as:

$$\frac{1-\beta}{\beta} \equiv z^* = \underset{(-)}{(1-\tau_k)^{\frac{1}{1-\beta}}} \; \underset{(?,\,+)}{z(\tau_k, \tau_c)^{**}}, \tag{16a}$$

while Condition $x^* = x^{**}$ is rewritten as:

$$\frac{\rho}{\sigma} + A\left(1 - \frac{\beta}{\sigma}\right)(z^*)^{1-\beta} - \left[\frac{A\beta}{\sigma}(z^*)^{1-\beta} - \frac{\rho}{\sigma}\right]\left\{1 + \left[\frac{A\beta}{\sigma}(z^*)^{1-\beta} - \frac{\rho}{\sigma}\right]\right\}z^* \equiv x^* \equiv \underset{(?,\,-)}{x(\tau_k, \tau_c)^{**}}. \tag{16b}$$

Relationships (16a)-(16b) impose two constraints on two tax instruments, implying the possible existence of a unique set of first-best optimal tax rates (τ_k^*, τ_c^*). To specifically determine the set of tax rates, we start from examining (16a). While its lefthand side is constant, the righthand side includes a direct negative effect of capital taxes, together with an indirect ambiguous effect of capital taxes, and a indirect positive effect of consumption taxes, both through equilibrium public to private capital ratio $z(\tau_k, \tau_c)^{**}$. To the extent that the direct negative effect of τ_k dominates the indirect ambiguous effect of τ_k, (16a) is a positively slopping locus in plane (τ_k, τ_c), with vertical intercept $\tau_c^0 > 0$, as illustrated by Locus $\gamma^* = \gamma^{**}$ in Figure 5.[14]

Next, for relationship (16b), both capital and consumption tax rates only have an indirect effect on the right-handed side, via equilibrium consumption to capital ratio $x(\tau_k, \tau_c)^{**}$: while consumption tax rates unambiguously reduce equilibrium consumption to capital ratios, with capital taxes having an ambiguous effect on equilibrium consumption to capital ratios. In the case where the effect of capital taxes on equilibrium $x(\tau_k, \tau_c)^{**}$ is negative, relationship (16b) is a negatively slopping locus in plane (τ_k, τ_c), with intercept $\tau_c^1 > 0$ as illustrated by Locus $x^*=x^{**}$ in upper Figure 5. On the other hand, when the effect of capital tax on equilibrium $x(\tau_k, \tau_c)^{**}$ is positive, (16b) is a positively slopping locus in plane (τ_k, τ_c), with intercept $\tau_c^1 > 0$, as illustrated by Locus $x^*=x^{**}$ in lower Figure 5.[15] Therefore, under proper parameter values that guarantee $\tau_c^1 > \tau_c^0$, intersections of Loci $x^*=x^{**}$ and $\gamma^* = \gamma^{**}$ determine a unique tax structure (τ_k^*, τ_c^*). Substituting into (11a)-(11b) and (12), leads to the equilibrium allocation of public to private capital ratios, consumption to private capital ratios and economic growth rates, that duplicate optimal allocation as determined in (8a)-(8b).

[Insert Figure 5 here]

How do the adjustment costs of public capital affect optimal tax structure? Given that a higher unit adjustment cost requires a higher average tax rate, according to (9), either a capital tax rate or a consumption

[14] Intercept τ_c^0 satisfies $z(0,\tau_c^0)^{**} = \frac{1-\beta}{\beta}$, where τ_c^0 decreases in β.

[15] Intercept τ_c^1 is determined by $z(0, \tau_c^1)^{**}=x^*$, where x^* is on the lefthand side of (16b). As equilibrium z^{**} decreases in τ_c, proper parameter values that reduce x^* to a certain level will lead to $\tau_c^1 > \tau_c^0$. The situation where a positively slopping Locus $x^*=x^{**}$ is steeper than Locus $\gamma^*=\gamma^{**}$ in lower Figure 5 is ruled out, because were this situation not ruled out, both τ_y and τ_c increase in productivity A, a result inconsistent with the prediction in (9).

tax rate must be higher. Which is most likely to increase? According to Locus $\dot{z}=0$ in Figure 2, a higher unit adjustment cost reduces steady-state equilibrium public to private capital ratios, allowing Locus $\gamma^* = \gamma^{**}$ to shift leftward (Figure 6). On the other hand, a higher unit adjustment cost has an ambiguous effect on steady-state equilibrium consumption to private capital ratios. The following possibilities emerge. (i) When Locus $x^*=x^{**}$ is negatively slopping as in upper Figure 6, it may shift upward or downward. Therefore, either both tax rates optimally increase (E_1), or the optimal consumption tax rate increases while the optimal capital tax rate decreases (E_2 and E_3). (ii) Alternatively, when Locus $x^*=x^{**}$ is positively slopping as in lower Figure 6, it shifts upward only,[16] so that optimal consumption tax rates must increase, but optimal capital tax rates may increase (E_1) or decrease (E_2).

[Insert Figure 6 here]

Therefore, higher unit adjustment costs increase optimal consumption tax rates, with ambiguous effects on optimal capital tax rates. Intuitively, higher unit adjustment costs capture the inefficiency of public capital formation, relative to private capital formation. As the capital taxation is like a user's fee, and an increase in the inefficiency of public capital is not due to an individual's increase in the use of public capital, the government optimally increases consumption taxes to finance the inefficiency. To summarize:

Proposition 5. *Under proper parameter values, there exists a unique interior optimal combination of capital and consumption tax rates, that achieves the first-best optimal allocation in a market economy. While a higher unit adjustment cost of public capital raises optimal consumption tax rates, it has an ambiguous effect upon optimal capital tax rates*.

To compare with conventional wisdom, our interior optimal capital and consumption tax rates differ

[16] A downward shift of Locus $x^*=x^{**}$ reduces both tax rates, which is inconsistent with (9).

from the Chamley (1986) proposition that asserts zero optimal capital tax rate in a neoclassical model. Moreover, without resorting to a positive congestion of public capital, our result of optimal positive capital taxation generalizes Turnovsky's (1996a, b) models, that reject the Chamley proposition, only under the condition that a positive congestion must be associated with the public capital. Our model generalizes Turnovsky (1996a, b) mainly because it allows for public investment to accumulate capital. As there is always a positive spillover from public capital in our model, given by initial and existing public capital stock, the capital taxation is like a user fee, internalizing the positive effects of public capital on marginal productivity of private capital. As a consequence, a positive capital tax rate is always necessary in optimum.

In our model, the government expenditure is optimally set, but the adjustment costs of public capital always increase optimal consumption tax rates with an ambiguous effect upon optimal capital tax rate. This result stands in sharp contrast to the zero effect of the adjustment costs of private capital, upon optimal tax structure between capital and consumption, when government expenditure is optimally set in Turnovsky (1996b). The different result lies in the characteristics of adjustment costs. For the adjustment cost of private capital, private sectors naturally internalize it, and thereby optimal taxation does not need to respond to it. However, private sectors do not react to the adjustment costs of public capital, generated by the government. As a result, the government needs to internalize it, and thus optimal taxation must be responsive to the adjustment costs of public capital. Moreover, as the capital taxation is like a user's fee, the government therefore, tends to use consumption taxes to finance the adjustment costs.

V. Concluding Remarks

This paper studies the optimal tax structure between capital and consumption taxation, in a dynamic AK model with public capital. One essential nature of the models with public capital is an interdependent treatment between government's taxation and expenditure. Our model differentiates itself from existing similar studies in that it allows for public investment to accumulate capital stock, and considers the adjustment costs of public capital formation. While the former feature allows for an otherwise long-run AK model to exhibit transitional dynamics, the latter feature links the "Tobin q" theory of investment to public investment. Under this framework, we study both a command economy and a market economy. We characterize the market equilibrium in steady state and transitional dynamics, and derive optimal tax structure between capital and consumption.

We find that, under proper parameter values, there is a unique interior tax structure for capital and consumption taxation, so that the market equilibrium can support the first-best optimal allocation. While our model changes the zero optimal capital taxation argument proposed by Chamley, it does not rely on any congestion in the use of public capital. We also find that although the adjustment cost of public capital reduces *equilibrium* economic growth in transitions and steady states, it affects neither the first-best *optimal* public to private capital ratios, nor the first-best *optimal* economic growth. However, the adjustment cost raises the optimal average tax rate through increasing optimal consumption tax rates, albeit its effect upon optimal capital tax rates is ambiguous.

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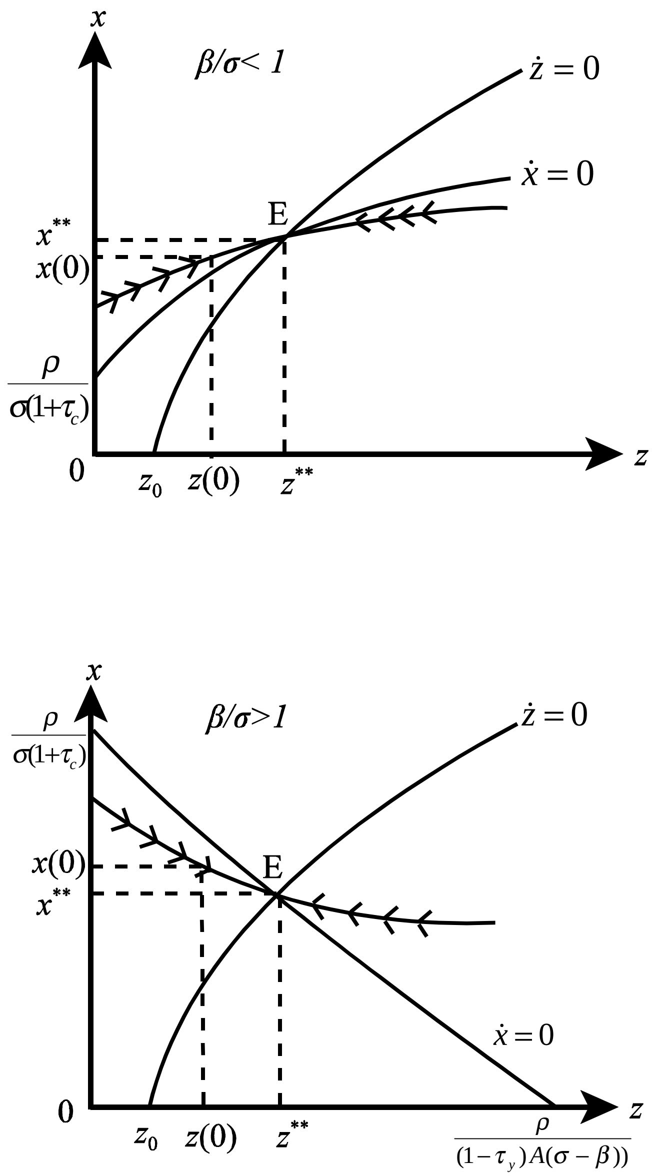


Figure 1. Market Equilibrium and Transitional Dynamics

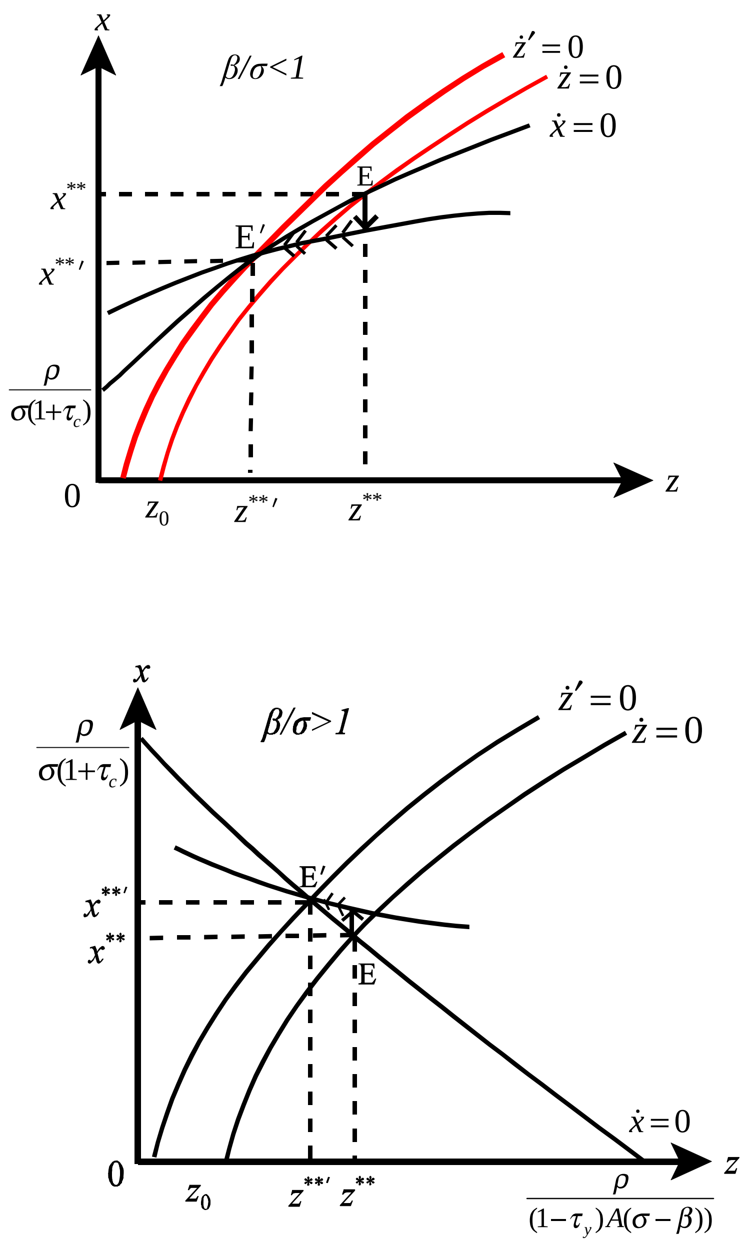


Figure 2. A High Capital Adjust Cost

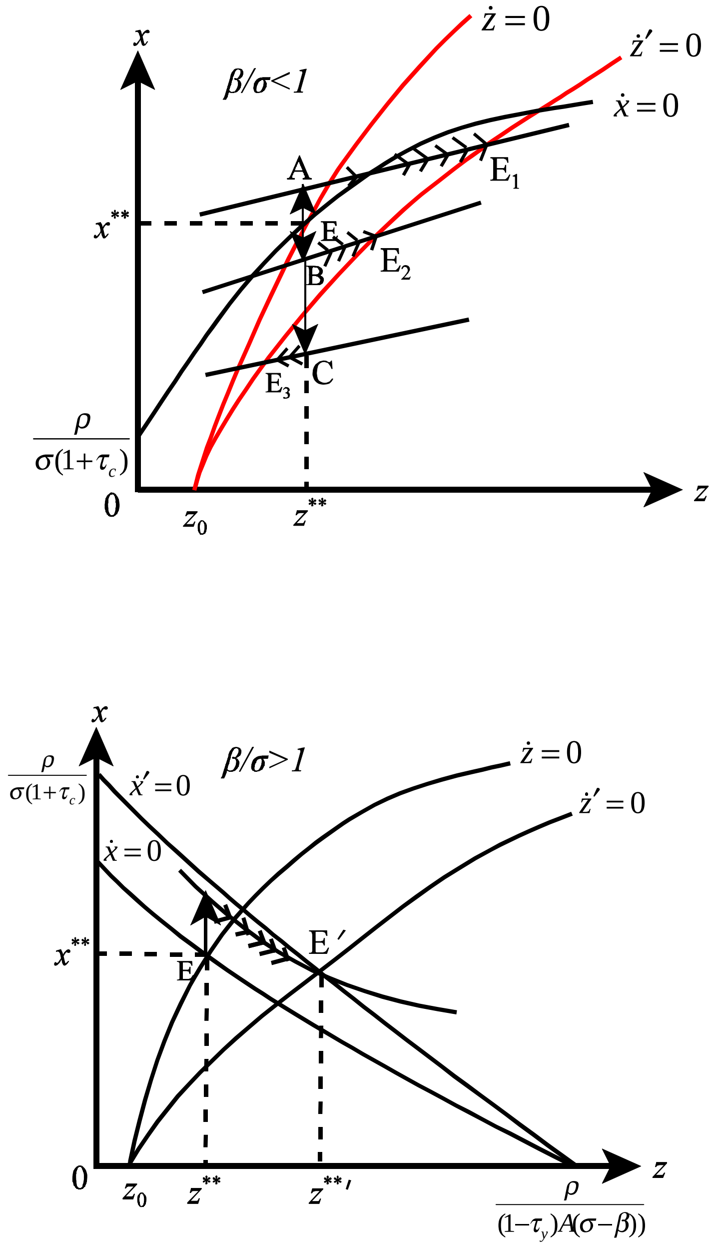


Figure 3. A Higher Capital Tax Rate

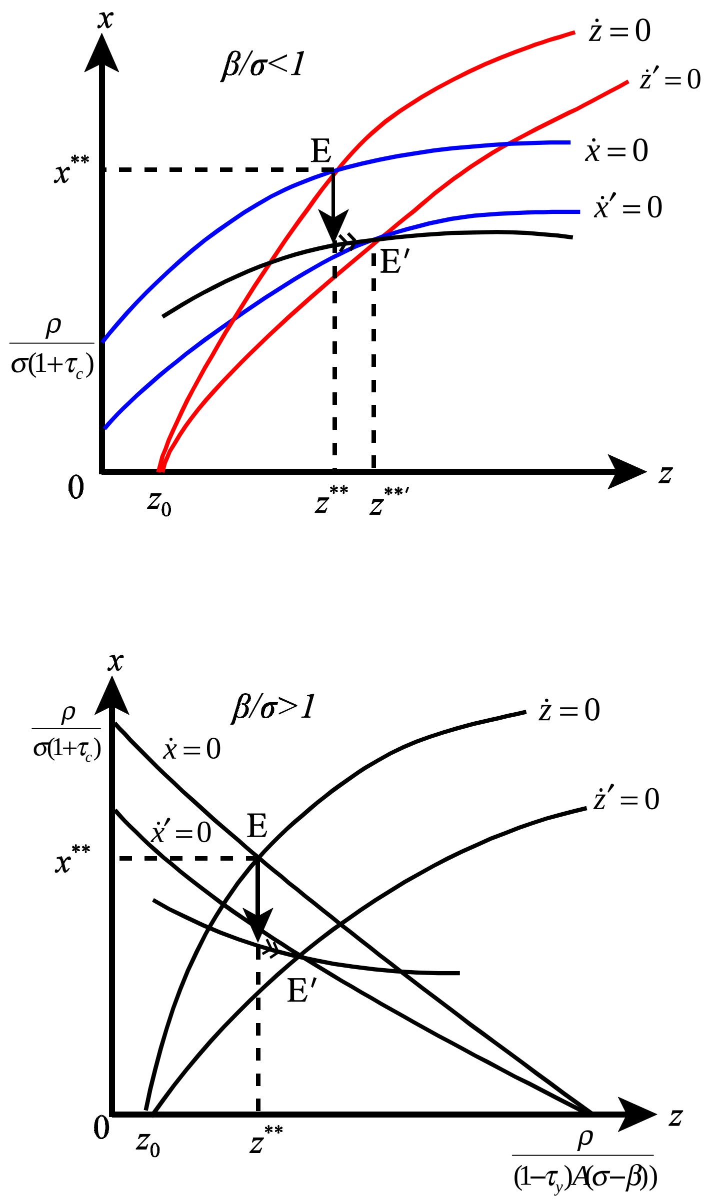
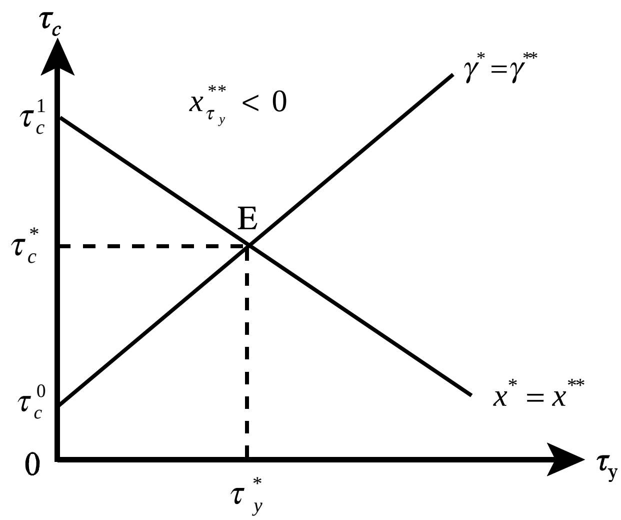


Figure 4. A Higher Consumption Tax Rate



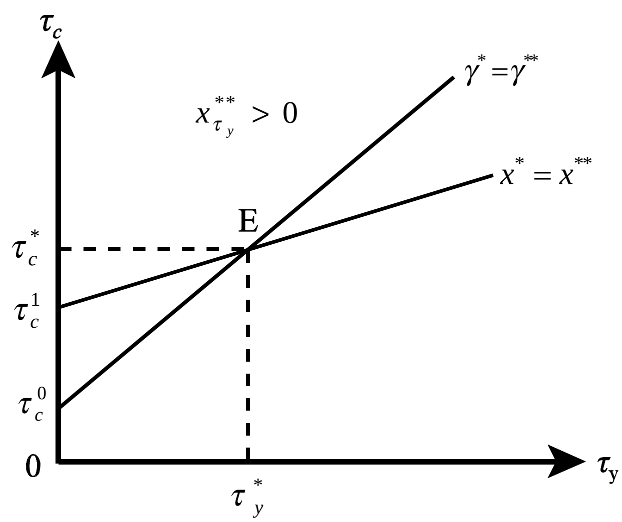
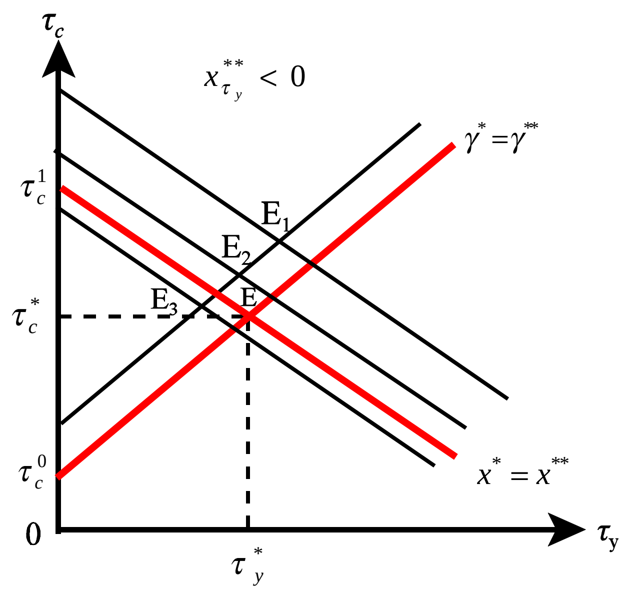


Figure 5. First-Best Optimal Tax Structure



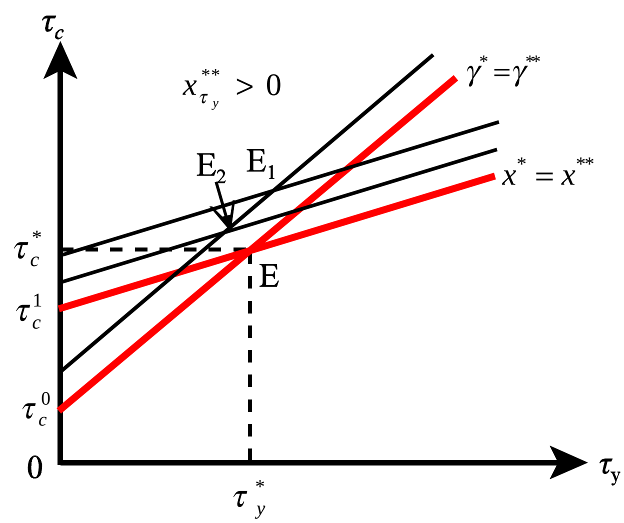


Figure 6. A Higher Capital Adjustment Cost

Appendix (Not Intended for Publication)

I Comparative Statics of Market Equilibrium

Totally differentiating (11a)–(11b) around a steady state leads to:

$$\begin{pmatrix} \frac{a_{11}}{x^{**}} & \frac{a_{12}}{x^{**}} \\ \frac{a_{21}}{z^{**}} & \frac{a_{22}}{z^{**}} \end{pmatrix} \begin{pmatrix} dx \\ dz \end{pmatrix} = \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} d\tau_k + \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix} d\tau_c + \begin{pmatrix} b_{13} \\ b_{23} \end{pmatrix} d\varphi, \qquad \textbf{(A1)}$$

where a_{11}, a_{12}, a_{21}, a_{22}, and Φ are as defined in (13), $b_{11} = -A(1-\frac{\beta}{\sigma})z^{-\beta} \begin{Bmatrix} < \\ > \end{Bmatrix} 0 \;\; if \;\; \beta/\sigma \begin{Bmatrix} < \\ > \end{Bmatrix} 1$, $b_{21} = -z^{-\beta-1} - A\frac{1}{\Phi}z^{-\beta} < 0$, $b_{12} = -x < 0$, $b_{22} = -x - \frac{1}{\Phi}z^{-1}x < 0$, and $b_{23} = \frac{1}{\varphi^2}\left(\Phi - 1 - \frac{\varphi(\tau_k A z^{-\beta} + \tau_c z^{-1}x)}{\Phi}\right) > 0$.

Note that Φ>1 if ϕ>0, and Φ=1 if ϕ=1. Denote J as the Jacobian matrix in (A1). Recall that a unique saddle path requires:

$$Det(J) = a_{11}a_{22} - a_{21}a_{12} = -(1+\tau_c)[\beta\tau_k\Phi z^{**-1-\beta} + (1-\tau_k)A(1-\beta)z^{**-\beta}] + [(1+\tau_c) + \tau_c\Phi z^{**-1}]A[(1-\beta)(1-\tau_k)(1-\frac{\beta}{\sigma})]z^{**-\beta} < 0.$$

When ϕ=0, the above requirement reduces to:

$$Det(J)\big|_{\varphi=0} = -(1+\tau_c)[\beta\tau_k z^{**-1-\beta} + (1-\tau_k)A(1-\beta)z^{**-\beta}] + [(1+\tau_c) + \tau_c z^{**-1}]A[(1-\beta)(1-\tau_k)(1-\frac{\beta}{\sigma})]z^{**-\beta} < 0.$$

1. The effect of a higher adjustment cost:

$$\frac{dz^{**}}{d\varphi} = \frac{a_{11}b_{23}}{Det(J)} = \frac{(+)(+)}{(-)} < 0, \quad \frac{dx^{**}}{d\varphi} = \frac{-a_{12}b_{23}}{Det(J)} = \frac{-\begin{pmatrix} - \\ + \end{pmatrix}(+)}{(-)} \begin{Bmatrix} < \\ > \end{Bmatrix} 0 \;\; if \;\; \beta/\sigma \begin{Bmatrix} < \\ > \end{Bmatrix} 1.$$

2. The effect of a higher (capital) income tax rate:

$$\frac{dz^{**}}{d\tau_k} = \frac{a_{11}b_{21} - a_{21}b_{11}}{Det(J)} = \frac{(+)(-) - (+)\begin{pmatrix} (-) \\ (+) \end{pmatrix}}{(-)} \begin{Bmatrix} ? \\ > \end{Bmatrix} 0 \;\; if \;\; \beta/\sigma \begin{Bmatrix} > \\ < \end{Bmatrix} 1.$$

$$\frac{dx^{**}}{d\tau_k} = \frac{b_{11}a_{22} - b_{21}a_{12}}{Det(J)} = \frac{\begin{pmatrix}(-)\\(+)\end{pmatrix}(-) - (-)\begin{pmatrix}(-)\\(+)\end{pmatrix}}{(-)} \begin{Bmatrix}?\\?\end{Bmatrix} 0 \ \ if \ \ \beta/\sigma \begin{Bmatrix}>\\<\end{Bmatrix} 1.$$

$$\left.\frac{dz^{**}}{d\tau_k}\right|_{\varphi=0} = \frac{a_{11}b_{21} - a_{21}b_{11}}{Det(J)|_{\varphi=0}} = \frac{(+)(-) - (+)\begin{pmatrix}(-)\\(+)\end{pmatrix}}{(-)} \begin{Bmatrix}?\\>\end{Bmatrix} 0 \ \ if \ \ \sigma \begin{Bmatrix}>\\<\end{Bmatrix} \beta.$$

The growth effect of a higher income tax rate is ambiguously affected by the adjustment costs of capital, even though the absolute value of the denominator in $\frac{dz^{**}}{d\tau_k}$, is larger than its counterpart in $\left.\frac{dz^{**}}{d\tau_k}\right|_{\varphi=0}$. This is because the absolute value of the first term in the numerator of $\frac{dz^{**}}{d\tau_k}$ is also larger than its counterpart in $\left.\frac{dz^{**}}{d\tau_k}\right|_{\varphi=0}$.

3. The effect of a higher consumption tax rate:

$$\frac{dz^{**}}{d\tau_c} = \frac{a_{11}b_{22} - a_{21}b_{12}}{Det(J)} = \frac{-x^{**}/z^{**}}{(-)} > 0,$$

$$\frac{dx^{**}}{d\tau_c} = \frac{b_{12}a_{22} - b_{22}a_{12}}{Det(J)} = \frac{\frac{Axz^{-\beta}}{\Phi}[\beta\tau_k - zx(1-\beta)(1-\tau_k)(1-\frac{\beta}{\sigma})] + (1-\tau_k)A(1-\beta)xz^{\beta}[z - x(1-\frac{\beta}{\sigma})]}{(-)}$$

$$\begin{Bmatrix}<\\<\end{Bmatrix} 0 \ \ if \ \ \begin{Bmatrix} \frac{\beta}{\sigma}<1, \ x^{**2} < \frac{\tau_k}{1-\beta}\frac{\beta}{1-\tau_k} \\ \beta/\sigma > 1 \end{Bmatrix}.$$

$$\left.\frac{dz^{**}}{d\tau_c}\right|_{\varphi=0} = \frac{-x^{**}/z^{**}}{Det(J)|_{\varphi=0}} = \frac{-x^{**}/z^{**}}{(-)} > 0.$$

When $\beta/\sigma<1$, to derive $\frac{dx^{**}}{d\tau_c}<0$ requires (i) $\frac{\tau_k}{1-\beta}\frac{\beta}{1-\tau_k} > zx(1-\frac{\beta}{\sigma})$ and (ii) $\frac{z^{**}}{x^{**}} > 1-\frac{\beta}{\sigma}$, in order to obtain a positive numerator. These two conditions imply the requirement of $x^{**2} < \frac{\tau_k}{1-\beta}\frac{\beta}{1-\tau_k}$, which is easy to meet, as x is much smaller than 1, while τ_k is usually about 1-β.

As $\Phi>1$ when $\phi>0$, the absolute value of the denominator in $\frac{dz^{**}}{d\tau_c}$, is larger than that in $\left.\frac{dz^{**}}{d\tau_c}\right|_{\varphi=0}$. Therefore, $\frac{dz^{**}}{d\tau_c} < \left.\frac{dz^{**}}{d\tau_c}\right|_{\varphi=0}$. Public capital adjustment costs reduce the positive effect of larger consumption tax rates on public to private capital ratios.